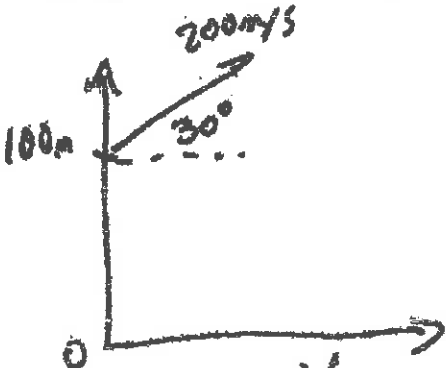
Practice Final Solutions

① Godzilla vs. Hulk.



when $y=0$, $x=?$

$$x = V_0 (\cos 30^\circ) t = (173\,\text{m/s})\,t$$

$V_0 = 200\,\text{m/s}$, $\cos 30^\circ \approx 0.866$

$g = 10\,\text{m/s}^2$ just use

$$y = y_0 + V_0 \sin 30^\circ\, t + \frac{1}{2} a_y t^2$$

$$y = 100\,\text{m} + (200\,\text{m/s})(0.5)t + \frac{1}{2}(-10\,\text{m/s}^2)t^2$$

$$0 = y = 100\,\text{m} + (100\,\text{m/s})t - 5\,\text{m/s}^2\, t^2$$

$a = -5 \quad b = 100 \quad c = 100$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-100 \pm \sqrt{10000 + 2000}}{-10}$$

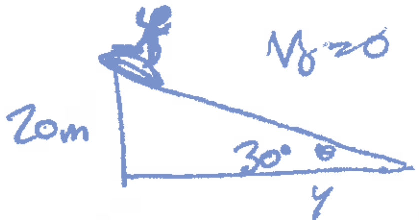
$t = +21$ sec.

$$\boxed{x = (173\,\text{m/s}) \cdot 21.0 = 3633\,\text{m}}$$

~ 3.6 km

w/ sig figs.

6. Capt. America chases Hydra agent

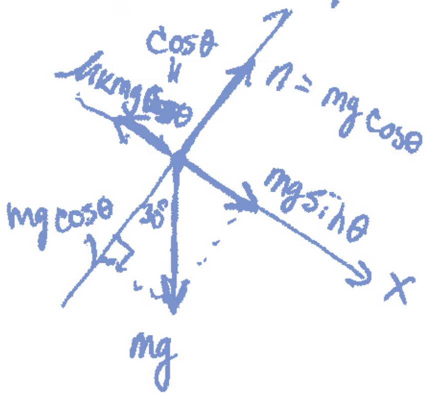


$v_0 = 0$

$m_{CA} = 100\ kg$

$\mu_K = 0.2$

$g \downarrow 10\ m/s^2$



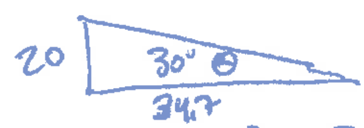
$\sum F_y = 0 \ , \ w_y = n$

X: $\sum F_x = ma$

$\cancel{m} \overset{10}{g} \underset{0.5}{\sin\theta} - \underset{0.2}{\mu_K} \cancel{m} \overset{10}{g} \underset{0.866}{\cos\theta} = \cancel{m} a_x$

$\underline{a = 3.27\ m/s^2}$ along rail

Rail = hypotenuse $= \sqrt{\underset{34.7}{x}^2 + \underset{20m}{y}^2}$



$\underset{0.577}{\tan 30°} = \frac{20}{Adj.}$

Rail = 40 m

$40\ m = \frac{1}{2} \overset{3.27}{a} t^2$

$\boxed{t = 4.9\ s}$

③ Wonder Woman vs. Cheetah

$m_{ww} = 80\,kg$ $m_c = 60\,kg$

$V_{ww} = 10\,m/s$ $V_c = 8\,m/s$

Inelastic collision. Momentum conserved. Energy Not.

$$p = m_{ww} \cdot V_{ww} + m_c V_c = (m_{ww} + m_c) V_{final}$$

$$\underset{80}{\shortparallel} \quad \underset{10}{\shortparallel} \quad \underset{60}{\shortparallel} \quad \underset{8}{\shortparallel} \quad \underset{140\,kg}{\shortparallel} \quad \underset{?}{\shortparallel}$$

$$1280\ kg \cdot m/s = 140 \cdot V_{final}$$

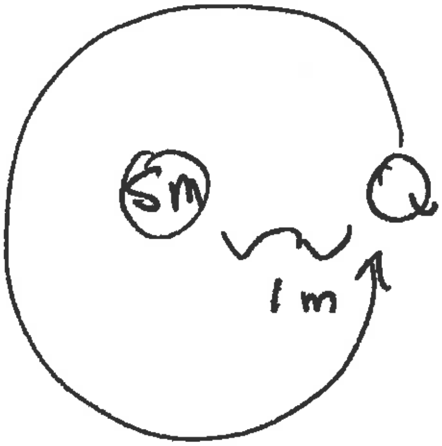
$$\boxed{V_{final} = 9.1\ m/s}$$

$$\frac{K_1}{K_2} = \frac{\cancel{\tfrac{1}{2}} m_{ww} V_{ww}^2 + \cancel{\tfrac{1}{2}} m_c V_c^2}{\cancel{\tfrac{1}{2}} (m_{ww} + m_c) V_{final}^2} = \frac{8000\,J + 3840\,J}{(140)(9.1)^2\ J}$$

$$= \frac{11840\ \cancel{J}}{11593\ \cancel{J}}$$

$$= \boxed{1.02}$$

④ Quicksilver Hits Spider-man



$a = 10\,m/s^2$ for $t = 10s$.

5 hits per circle.

$m_Q = 70\,kg$

a. What is final speed in m/s? $\boxed{100\,m/s}$

b. Final Kinetic Energy? $\boxed{350\,kJ}$

c. Final angular speed ω? $\boxed{100\,rad/sec}$

d. Number of hits total? 398

$$a = \frac{dv}{dt} \quad \text{So} \quad V_f = \underset{0}{V_0} + \underset{10\,m/s^2}{a}\,\underset{10}{t} = \boxed{100\,m/s}$$

constant.

$$K = \frac{1}{2} m_Q V_Q^2 = \frac{1}{2}(70\,kg)(100\,m/s)^2 = 350{,}000\,J$$

Not a rigid rotator, not spinning! Only $\frac{1}{2} m v_Q^2$.

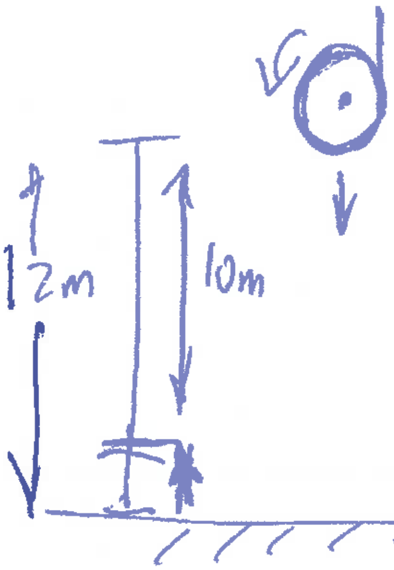
$$V = r\omega \quad \text{so} \quad \omega = \frac{V_{final}}{r = 1m} = 100\,rad/s$$

$$\#\text{ Hits} = \left(\frac{\text{total distance}}{2\pi r}\right) \times 5$$

$$x = \cancel{x_0}^{\,0} + \cancel{v_0}^{\,0} t + \frac{1}{2} a t^2 = \boxed{500\,m}$$

$$= \left(\frac{500\,\cancel{m}}{2\pi\,\cancel{m}}\right) 5 = 398$$

⑤. Joker's Giant yo-yo trap



$m = 100\,kg$

$r = 1\,m$

$g \downarrow = 10\,m/s^2$

$V_f = ?$

$I = \frac{1}{2} M R^2$; $M = 100kg$; $R = 1m$

$t = 0$, $V_0 = 0$, $\omega = 0$

$$U_1 + K_1 = U_2 + K_2$$

$U_1 = mgh$ ($m = 100kg$, $g = 10\,m/s^2$, $h = 10m$); $K_1 = 0$; $U_2 = 0$; $K_2 = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega^2$; $V = R\omega$, $\omega = \frac{V}{R}$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}V_f^2 + \frac{1}{2}\cancel{m}\cancel{R^2} \cdot \frac{V^2}{\cancel{R^2}} \cdot \frac{1}{2}$$

$$g \cdot 10m = \frac{1}{2}V_f^2 + \frac{1}{4}V_f^2 = \frac{3}{4}V_f^2 = 100\,m^2/s^2$$

$$V_f^2 = \frac{400}{3}\,m^2/s^2$$

$$\boxed{V_f = 11.5\,m/s}$$

⑥ Vision flies!

$Vol. = 0.1\ m^3$

$F_B = \rho_{Air}\, g\, \underset{\rightarrow 0.1}{Vol.}$, $\rho_{Air} = 1.2\ kg/m^3$

$$\Sigma F_y = m_v a$$

up is positive

$$(\rho_{Air}\, g\, Vol.) - m_v g = m_v a$$

$$m_v = \frac{1}{2} \rho_{air} Vol.$$

$$\cancel{\rho_{Air}}\, g\, \cancel{Vol.} - \frac{1}{2} \cancel{\rho_{air} Vol.}\, g = \frac{1}{2} \cancel{\rho_{air} Vol.}\, a$$

$$\frac{\cancel{1}}{2} g = \frac{\cancel{1}}{\cancel{2}} a$$

$$\boxed{a = g \approx 10\ m/s^2 \text{ up}}$$

$$V = \underset{0}{V_0} + \underset{10\ m/s^2}{a}\, \underset{\rightarrow 5}{t}$$

$$\boxed{V = 50\ m/s}$$

$$y = \underset{0}{y_0} + \underset{0}{v_{oy}} t + \frac{1}{2} \underset{10\ m/s^2}{a_y} \underset{\rightarrow 25\ s^2}{t^2}$$

$$y = \frac{250}{2}\ m = \boxed{125\ m}$$

7. Geosynchronous JLA Watchtower

$$T = P = 1 \text{ day} = 86\,400 \text{ sec}$$

$$M_E = 5.97 \times 10^{24} \text{ kg}$$

Kepler $$T = 2\pi r^{3/2} / \sqrt{G M_E}$$

$$G = 6.67 \times 10^{-11} \ N \cdot (m/kg)^2$$

$$r^{3/2} = \frac{T}{2\pi} \sqrt{G M_E}$$

$$r = \left(\underbrace{\frac{86400 s}{2\pi}}_{= 13800} \underbrace{\sqrt{6.67 \times 10^{-11} \times 6 \times 10^{24}}}_{\sqrt{39.8 \times 10^{13}} \,=\, 1.99 \times 10^{8}} \right)^{2/3}$$

$$r = \left(2.75 \times 10^{12} \right)^{2/3} \text{ m}$$

$$\boxed{r = 4.27 \times 10^{8} \text{ m} = 42{,}700 \text{ km}}$$

(26 500 miles)

Note: often altitude is given above sea level, $R_E \approx 6400$ km.