Practice Exam 2 Fall 2017 (Brotherton)

Phys 1210 (Ch. 5-8)

____

your name

The exam consists of 6 problems. Each problem is of equal value.

You can skip one of the problems (best five will count if you do all problems). Calculators are allowed.

Tips for better exam grades:

Read all problems right away and ask questions as early as possible.

Make sure that you give at least a basic relevant equation or figure for each sub-problem.

Make use of the entire exam time. When you are done with solving the problems and there is some time left, read your answers over again and search for incomplete or wrong parts.

Show your work for full credit. The answer '42' only earns you any credit IF '42' is the right answer. We reserve points for 'steps in between', figures, units, etc.

No credit given for illegible handwriting or flawed logic in an argument.

Remember to give units on final answers.

Please box final answers so we don't miss them during grading.

Please use blank paper to write answers, starting each problem on a new page.

Please use 10 m/s^2 toward the center of the Earth as the acceleration due to gravity on Earth, but use two significant figures for such problems

'Nuff said.

1. **Batman Saves Batgirl and Robin**
Following the explosion of a bobby-trapped Jack-in-the-Box left by the Joker, Batman (120 kg) is standing on top of a building holding onto a spring from which dangles Batgirl (60 kg). She in turn is holding another spring from which dangles Robin (50 kg). If the spring constant k = 5 N/mm for both springs, by what lengths are the two springs stretched? Again, use g = 10 m/s^2 and provide your answer in meters.

2. **Iceman slides into battle!** Iceman (80kg) slides down an ice slide into a fight against Magneto. Assume his slide is straight and makes a 30 degree angle with respect to the ground. Assume a coefficient of kinetic friction of 0.05 between Iceman and his slide, and that he starts from rest 30 meters above the ground. What is his final velocity in his direction of motion? How long does it take him to reach the ground? Use g = 10 m/s^2 down and give your answer to two significant figures.

3. **Spider-man swings on his webline.**
Spider-man is standing at the top of a building 150 meters high. He shoots his web toward a flag pole 20 meters lower than his height on the side of another building. He jumps off the building and uses the taught webline to swing away. If the webline is 50 meters long, how fast is he moving in m/s at the bottom of his swing? Ignore air resistance and friction, and use g =10 m/s^2.

4. **Catwoman Drags Batman.** Batman (100 kg) has caught up to Catwoman (60 kg) on top of a building with a rain-slick roof you can treat as frictionless. She is 5 meters away from Batman, catches him with her whip, then steps off the edge. She falls, pulling Batman toward the edge. How fast is he sliding when he gets to the edge? (Don't worry – he'll cut the whip with a batknife just before he falls!)

5. **Ghost Rider in a Ring.**
Ghost Rider takes his motor cycle onto the track of a roller coaster that has a vertical ring, also known as a loop the loop. Assume that he's on the planet Earth, the mass of him and the bike is 400 kg, and the radius of the ring is 10 meters. He rides at a speed of 30 m/s on the inside of the ring. What is the normal force of the ring on the bike at (1) the bottom of the ring and (2) the top of the ring?

6. **Black Widow's guns.**
In the Avengers movie, the Black Widow uses a pair of Glock 26s. The mass of an unloaded Glock 26 is 615 grams (they're small!). The mass of one type of 9mm bullet used in the Glock 26 is 7.5 grams. The velocity of this bullet shot from the Glock 26 is about 370 m/s. What is the recoil velocity of a Glock 26 firing its last bullet in m/s? What is the ratio of the kinetic energy in the bullet to that of the recoiling gun?

Master Equations – Physics 1210

One-dimensional motion with constant acceleration:

① $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ find the other forms of master equation 1 by

(a) building the derivative of the equation
(b) solving the new equation for t and substituting it back into the master equation, and
(c) using the equation for average velocity times time

Two-dimensional motion for an object with initial velocity v_o at an angle α relative to the horizontal, with constant acceleration in the y direction:

② $x = x_0 + v_0 cos\alpha\, t$

③ $y = y_0 + v_0 sin\alpha\, t + \frac{1}{2}a_y t^2$ find the related velocities by building the derivatives of the equations

Newton's Laws

④ $\Sigma\vec{F} = 0,\ \Sigma\vec{F} = m\,\vec{a},\ \vec{F}_{A\to B} = -\vec{F}_{B\to A}$ find the related component equations by replacing all relevant properties by their component values

The quadratic equation and its solution:

$$a \cdot x^2 + b \cdot x + c = 0\,, then\ x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Table with some values for trig functions:

Degrees:	30	45	60	330	
sin	0.5	0.707	0.866	-0.5	
cos	0.866	0.707	0.5	0.866	
tan	0.577	1	1.732	-0.577	

Friction:

F = - μ_k * n (kinetic)
F < - μ_s * n (static upper limit)

Work and Power definitions:

Work $W = \vec{F} \cdot \vec{s} = Fs \cos \phi$
Power P = dW/dt

Hook's Law:

F = -kx (where k is the spring constant)

Kinetic Energy:

K = ½ mv^2 (linear)
K = ½ I w^2 (rotational)

Potential Energy:

U = mgh (gravitational)
U = ½ kx^2 (elastic for a spring constant k)

Work-energy with both kinetic and potential energy:

$K_1 + U_1 + W_{other} = K_2 + U_2$

Linear Momentum:

$$\vec{p} = m\vec{v} \text{ and } \vec{F} = d\vec{p}/dt$$

Impulse and Impulse-Momentum Theorem:

$$\vec{J} = \int_{t1}^{t2} \sum \vec{F}\, dt = \vec{p}_2 - \vec{p}_1$$

Angular-Linear Relationships:

a = v^2/r (uniform circular motion)
$v = r\omega$, $a_{tan} = r\alpha$, and $a_{rad} = v^2/r = r\omega^2$

Parallel axis theorem for the moment of inertia I:

$I_p = I_{cm} + Md^2$

Angular dynamics:

Torque $\vec{\tau} = \vec{r} \, X \, \vec{F}$ and $\sum \tau_z = I\alpha_z$

Angular Momentum:

$$\vec{L} = \vec{r} \, X \, \vec{p} \;\; and \; \vec{\tau} = d\vec{L}/dt$$

Center of Mass:

$$\vec{r}_{cm} = \frac{\sum_i m_i r_i}{\sum_i m_i}$$

Fluid Mechanics

$p = p_0 + \rho gh$ (pressure in an incompressible fluid of constant density)

$A_1v_1 = A_2v_2$ (continuity equation, incompressible fluid)

$dV/dt = Av$

$p_1 + \rho gy_1 + ½\, \rho v_1^2 = p_2 + \rho gy_2 + ½\, \rho v_2^2$ (steady flow, ideal fluid)

Gravity:

$F = Gm_1m_2/r^2$

$U = -Gm_Em/r$

T (orbital period) = $2\,\pi\, r^{3/2}$/sqrt(Gm_E)

$G = 6.67x10^{-11}$ N·(m/kg)^2

Periodic Motion

f = 1/T; T= 1/f

ω = 2π f = 2π/T (angular frequency here)

ω = sqrt(k/m) (k is spring constant)

x = A cos(ωt + Φ)

ω = sqrt (κ/I) (angular harmonic motion)

ω = sqrt (g/L) (simple pendulum)

ω= sqrt (mgd/I) (physical pendulum)

Mechanical Waves in General

V = λf

Y(x,t) = A cos (kx- ωt) (k is wavenumber, k = 2 πf)

V = sqrt (F/μ)

P_{av} = ½ sqrt(μF) $\omega^2 A^2$

$I_1/I_2 = (r_2/r_1)^2$ (inverse square law for intensity)

Sound Waves

P_{max} = BkA (B is bulk modulus)

B = (10 dB) log(I/I_0) where $I_0 = 1x10^{-12}$ W/m^2

$f_L = f_s * (v+v_L)/(v+v_s)$ -- Doppler effect

Table 9.1 Comparison of Linear and Angular Motion with Constant Acceleration

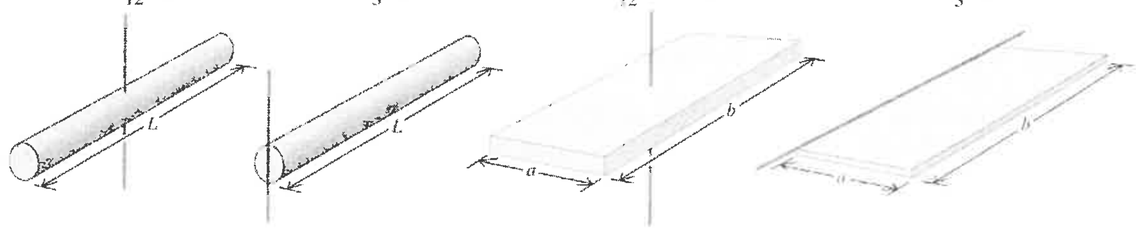
Straight-Line Motion with Constant Linear Acceleration	Fixed-Axis Rotation with Constant Angular Acceleration
$a_x = \text{constant}$	$\alpha_z = \text{constant}$
$v_x = v_{0x} + a_x t$	$\omega_z = \omega_{0z} + \alpha_z t$
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_x + v_{0x})t$	$\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$

(a) Slender rod, axis through center: $I = \frac{1}{12}ML^2$

(b) Slender rod, axis through one end: $I = \frac{1}{3}ML^2$

(c) Rectangular plate, axis through center: $I = \frac{1}{12}M(a^2 + b^2)$

(d) Thin rectangular plate, axis along edge: $I = \frac{1}{3}Ma^2$



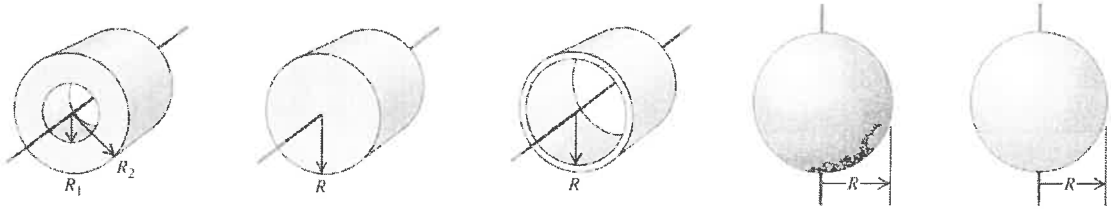
(e) Hollow cylinder: $I = \frac{1}{2}M(R_1^2 + R_2^2)$

(f) Solid cylinder: $I = \frac{1}{2}MR^2$

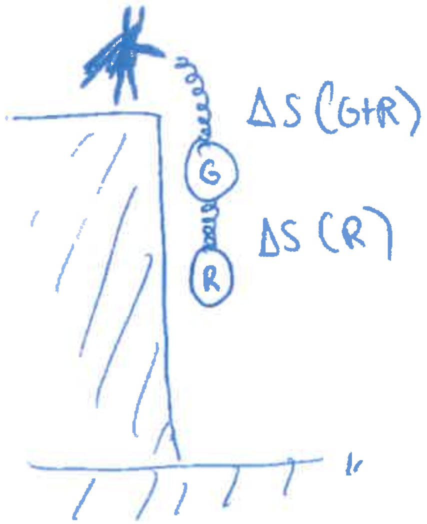
(g) Thin-walled hollow cylinder: $I = MR^2$

(h) Solid sphere: $I = \frac{2}{5}MR^2$

(i) Thin-walled hollow sphere: $I = \frac{2}{3}MR^2$



Bat family



Convert to mks:

$K = 5\ N/mm = 5000\ N/m$

ΔS in each Spring?

$|F| = k \cdot s$

$g = 10\ m/s^2$

$\downarrow$

Robin: $m_R = 50\,kg$

↑ T, ↓ mg — Static

$\Rightarrow |F| = T = mg = k \cdot \Delta S(R)$

$\Delta S(R) = \frac{mg}{k} = \frac{50 \times 10\,m}{5000} = \boxed{0.1\,m}$

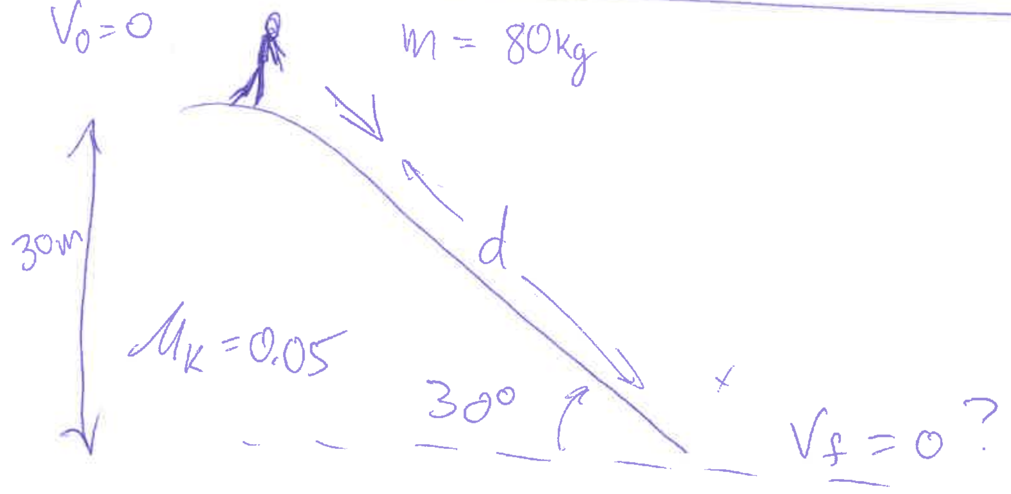
Robin + Batgirl: $m_{BG} = 60\,kg$

$\omega = (m_R + m_{BG})\,g$, with $m_R + m_{BG} = 110\,kg$ and $g = 10\,m/s^2$

$\Delta S(R+G) = \frac{110\,kg \cdot 10\,m/s^2}{5000\,N/m}$

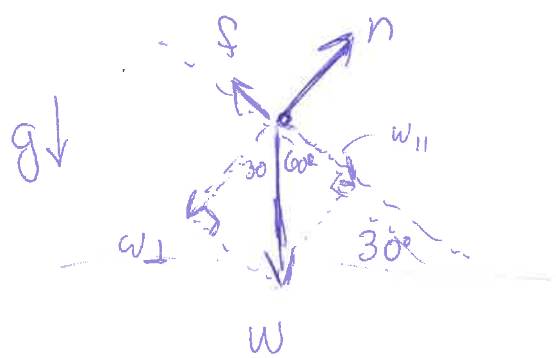
$\boxed{\Delta S(R+BG) = 0.22\,m}$

② Iceman Slides into Battle!



$g \downarrow 10\,\text{m/s}^2$

Can do with Newton's Laws or Work-Energy:



$w_{\parallel} = mg\sin 30° = 400\text{N}$

$w_{\perp} = mg\cos 30° = 693\text{N}$ =

$n - w_{\perp} = 0 \Rightarrow n = mg\cos 30°$

$\Rightarrow f = -\mu_K n$

$f = -34.6\,\text{N}$

Newton

$$\sum F_{\parallel} = w_{\parallel} + f = 400 - 34.6 = 365\text{N} = ma$$

(friction) $\Rightarrow a, \Rightarrow t$ etc. $\Rightarrow v$

Work energy

$$U_1 + K_1 + W_{other} = U_2 + K_2$$

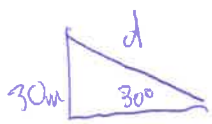
$$mgh_1 + \tfrac{1}{2}mv_1^2 + W_{other} = U_2 + K_2$$

($v_1 \to 0$, $U_2 \to 0$)

$$(80\,\text{kg})(10\,\text{m/s}^2)(30\,\text{m}) + (-34.6\text{N})(d) = \tfrac{1}{2} m V_f^2$$

(m = 80kg)

24000 J $\quad - 2076 \quad = \tfrac{1}{2}\, 80\text{kg}\, V_f^2$

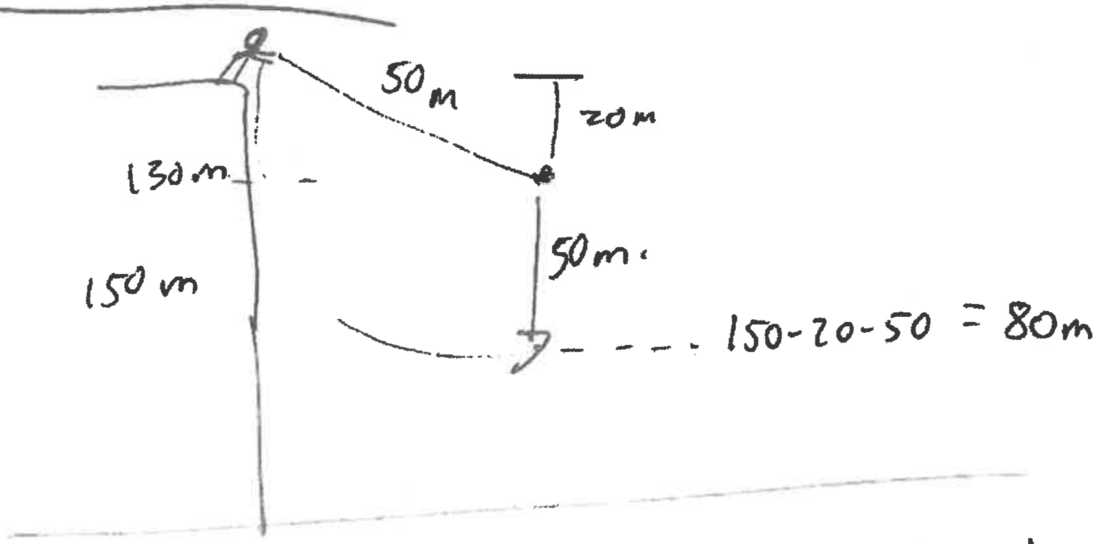


$$\sin\theta = \frac{O}{H} = \sin 30° = \frac{1}{2} = \frac{30\text{m}}{d} \Rightarrow d = 60\text{m}$$

$V_f = 23\,\text{m/s}$

Practice
Exam 2 Solutions

3.



Tension $\perp$ to motion $\Rightarrow$ No work.

$$K_1 + U_1 = K_2 + U_2$$

0 define=0 $\frac{1}{2}mv^2 + mgh$ (10, 70 m)

$$70\,m\,g = \frac{1}{2}v^2$$

$$(140\,m)(10)\frac{m}{s^2} = v^2$$

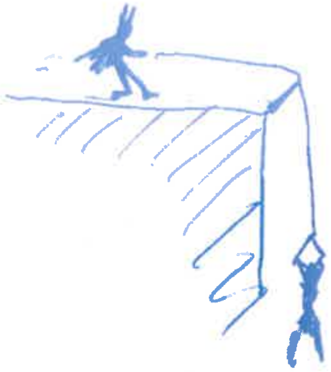
$$v = \sqrt{1400}\ m/s = \boxed{37\,m/s}$$

(82 mph!)

Catwoman drags Batman to Edge

(4) $m_B = 100\,kg$.

Rain-slick surface
Assume frictionless



$g \downarrow 10\,m/s^2$

$m_c = 60\,kg$.

$V_i = 0$ when Catwoman Steps off edge.

How fast is Batman Sliding after 5 m
When he hits the edge?

(Wwhip cancels out)

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

($W_{other} \to 0$; $K_1 = \frac{1}{2}mv_i^2$, $U_1 = mgh$)

Initially all zero on left.

$$0 = \frac{1}{2} m_B V_f^2 + \frac{1}{2} m_c V_f^2 - m_c g h$$

($\frac{1}{2}m_B$ = 50 kg, $\frac{1}{2}m_c$ = 30 kg, $h = -5\,m$, $-m_c g h$ = $-3000\,kg \cdot m^2/s^2$)

$$3000\,kg\,m^2/s^2 = 80\,kg\,V_f^2$$

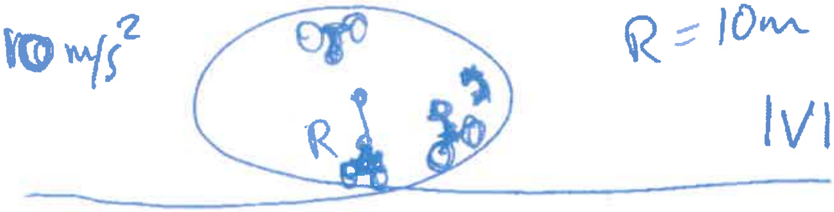
$$V_f^2 = (37.5)\,m^2/s^2$$

$$\boxed{V_f = 6.1\,m/s}$$

5. Ghost Rider in a Ring.

$m = 400\ kg$

$g \downarrow 10\ m/s^2$



$R = 10m$

$|V| = 30\ m/s$

FBD at Top.

Bottom:

$$|a_{rad}| = \frac{|V|^2}{R = 10m} = 30\ m/s$$

$$|a_{rad}| = \frac{900\ m^2/s^2}{10m} = 90\ m/s^2$$

Top

$$F = ma_{rad} = -w - n = m(-|a_{rad}|) = -400kg \cdot 90\ m/s^2$$

($w = mg$)

$$+(400kg)(10\ m/s^2) + n = +400kg \cdot 90\ m/s^2$$

$$n_{Top} = 36000N - 4000N = \boxed{32000N}$$

down.

Bottom

$$n - w = m|a_{rad}| = 36000\ N$$

($w = mg$)

$$n = 36000\ N + 4000N = \boxed{40000\ N}$$

($4000N = w$)

up.

Don't Try this at home!

Black Widow's Glock 22s

Apply conservation of momentum (No external impulses).

$$P_1 = P_2 = 0$$

$$P_2 = m_g V_g + m_B V_B = 0$$

m_g: 615g, V_g: Recoil Velocity, m_B: 7.5g, V_B: 370 m/s

Velocities in opposite directions.

$$|V_g| = \frac{m_B |V_B|}{m_g} = \frac{7.5g}{615g}(370 m/s)$$

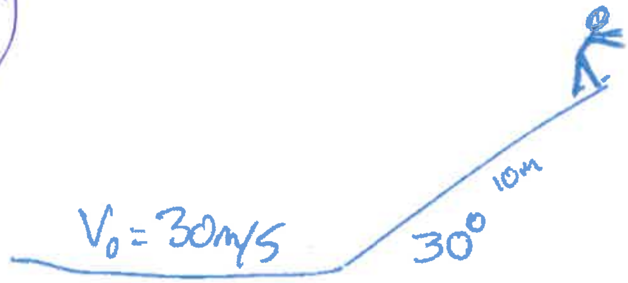
$$\boxed{V_g = 4.5 m/s}$$

Kinetic Energy Ratio:

$$\frac{K_B}{K_G} = \frac{\frac{1}{2} m_B V_B^2}{\frac{1}{2} m_g V_g^2} = \frac{7.5\,(370)^2}{615\,(4.5)^2}$$

$$\boxed{\frac{K_B}{K_G} \approx 82}$$

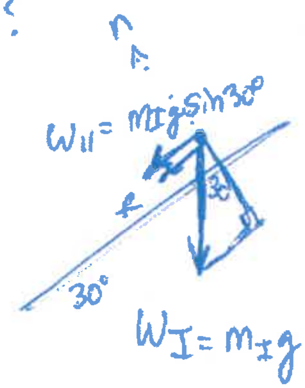
(2)



$\mu_K = 0.05$

$m_I = 70 kg$

$V_f = ?$



$f = n\mu_K = m_I g \cos\theta \, \mu_K$

$\sum F_{\parallel} = m_I a \Rightarrow -m_I g \sin 30° - m_I g \cos\theta \, \mu_k = m_I a$

$a = -g(0.5 + (0.87)(0.05))$ — $(0.87)(0.05) = 0.04$

$a = -10 m/s^2 (0.54)$

$a \simeq -5.4 m/s^2$

Const. a ∥ to ice:

$x = x_0 + V_{0x} t + \frac{1}{2} a_x t^2$

$10m = 0 + 30 m/s \, t - 5.4 m/s^2 \, t^2$

$-5.4 m/s^2 + 30 m/s \, t - 10m = 0$

Quad formula:

$t = 0.36$ sec. or 5.199 sec.

$V = V_0 + at$ — $V_0 = 30$, $a = -5.4$, $t = 0.36$; $at = 1.94 m/s$

$V_f \simeq 28 m/s$