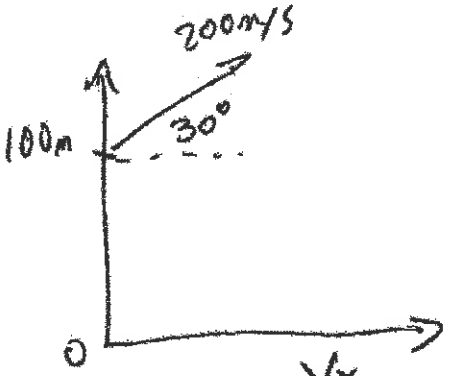
Practice Final Solutions

① Godzilla vs. Hulk.



when $y = 0$, $x = ?$

$x = V_0 (\cos 30^\circ) t = (173\ m/s)\, t$

(V_0 = 200 m/s, $\cos 30^\circ$ = 0.866)

$g = 10\ m/s^2$

just use

$y = y_0 + V_0 \sin 30^\circ t + \frac{1}{2} a_y t^2$

$y = 100m + (200\ m/s)(0.5)t + \frac{1}{2}(-10\ m/s^2)\, t^2$

$0 = y = 100m + (100\ m/s)t - 5\ m/s^2\, t^2$

$a = -5 \quad b = 100 \quad c = 100$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-100 \pm \sqrt{10000 + 2000}}{-10}$

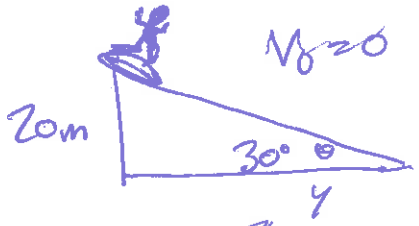
$t = +21$ sec.

$x = (173\ m/s) \cdot 21.0 = 3633m$

~ 3.6 km

w/ sig figs.

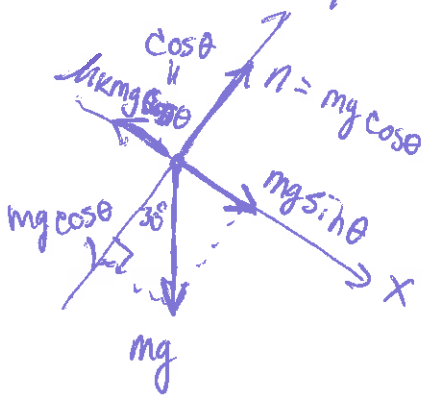
(10) Capt. America chases Hydra agent.



$m_{CA} = 100\ kg$

$\mu_K = 0.2$

$g \downarrow 10\ m/s^2$



$\sum F_y = 0$, $w_y = n$

X: $\sum F_x = ma$

$mg\sin\theta - \mu_K mg\cos\theta = ma_x$

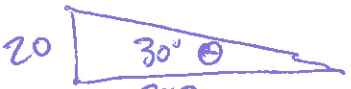
$10 \quad 10$

$\sin\theta = 0.5$, $\mu_K = 0.2$, $\cos\theta = 0.866$

$a = 3.27\ m/s^2$ along rail

Rail = hypotenuse $= \sqrt{x^2 + y^2}$

$x = 34.7$, $y = 20m$



$\tan 30° = \frac{20}{Adj.}$

0.577

Rail = 40 m

3.27

$40\ m = \frac{1}{2} a t^2$

$t = 4.9\ s$

(3) Wonder Woman vs. Cheetah

$m_{ww} = 80\,kg$ $\quad$ $m_c = 60\,kg$

$V_{ww} = 10\,m/s$ $\quad$ $V_c = 8\,m/s$

Inelastic collision. Momentum conserved. Energy Not.

$$p = m_{ww} \cdot V_{ww} + m_c V_c = (m_{ww} + m_c) V_{final}$$

$$80 \quad 10 \qquad 60 \quad 8 \qquad 140\,kg \qquad ?$$

$$1280\ kg \cdot m/s = 140 \cdot V_{final}$$

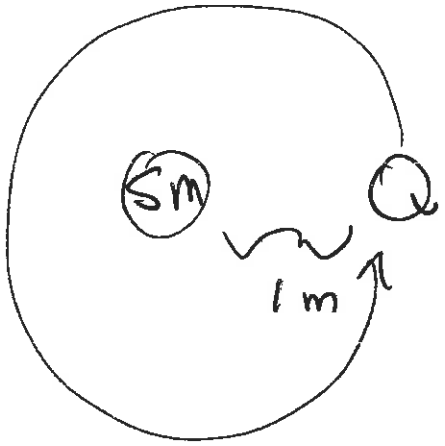
$$\boxed{V_{final} = 9.1\,m/s}$$

$$\frac{K_1}{K_2} = \frac{\frac{1}{2} m_{ww} V_{ww}^2 + \frac{1}{2} m_c V_c^2}{\frac{1}{2} (m_{ww} + m_c) V_{final}^2} = \frac{8000\,J + 3840\,J}{(140)(9.1)^2\,J}$$

$$= \frac{11\,840\,\cancel{J}}{11593\,\cancel{J}}$$

$$= \boxed{1.02}$$

(4) Quicksilver Hits Spider-man



$a = 10\ m/s^2$ for $t = 10s$.

5 hits per circle.

$m_Q = 70\ kg$

a. What is final speed in m/s? 100 m/s
b. Final Kinetic Energy? 350 kJ
c. Final angular speed ω? 100 rad/sec.
d. Number of hits total? 398

$a = \frac{dv}{dt}$ "constant." So $v_f = v_0 + at = 0 + 10\ m/s^2 \cdot 10 = 100\ m/s$

$K = \frac{1}{2} m_Q v_Q^2 = \frac{1}{2}(70kg)(100\ m/s)^2 = 350{,}000\ J$

Not a rigid rotator, not spinning! Only $\frac{1}{2}mv_Q^2$.

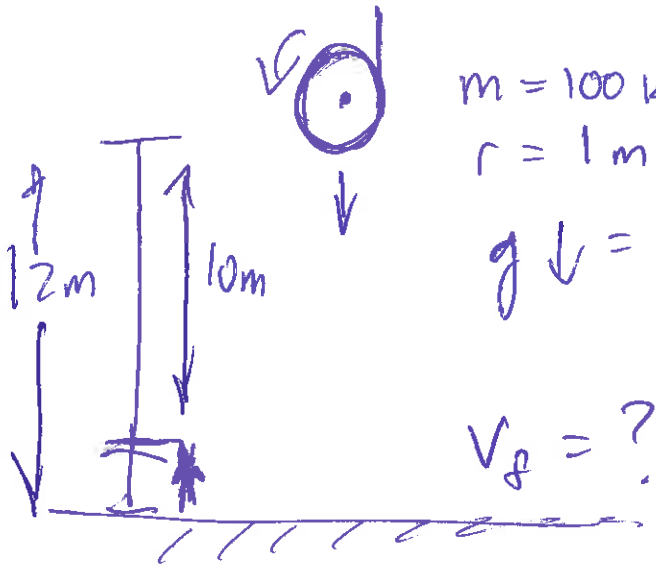
$v = r\omega$ so $\omega = \frac{v_{final}}{r = 1m} = 100\ rad/s$

$\#\ Hits = \left(\frac{\text{total distance}}{2\pi r}\right) \times 5$

$x = \cancel{x_0}^{\,0} + \cancel{v_0}^{\,0} t + \frac{1}{2}at^2 = 500\ m$

$= \left(\frac{500\ m}{2\pi\ m}\right) 5 = 398$

(5) Joker's Giant yo-yo trap



$m = 100\,kg$

$r = 1\,m$

$I = \frac{1}{2} M R^2$; $M = 100\,kg$, $R = 1\,m$

$g \downarrow = 10\,m/s^2$

$V_f = ?$

$t = 0$, $V_0 = 0$, $\omega = 0$

$U_1 + K_1 = U_2 + K_2$

$U_1 = mgh$; $h = 10\,m$, $m = 100\,kg$, $g = 10\,m/s^2$

$K_1 = 0$

$U_2 = 0$

$K_2 = \frac{1}{2} m V_f^2 + \frac{1}{2} I \omega^2$; $V = R\omega$, $\omega = \frac{V}{R}$

$$mgh = \frac{1}{2} m V_f^2 + \frac{1}{2} m R^2 \cdot \frac{V^2}{R^2} \cdot \frac{1}{2}$$

$$g \cdot 10m = \frac{1}{2} V_f^2 + \frac{1}{4} V_f^2 = \frac{3}{4} V_f^2 = 100\,m^2/s^2$$

$$V_f^2 = \frac{400}{3}\,m^2/s^2$$

$$\boxed{V_f = 11.5\,m/s}$$

(6) Vision flies!

$\text{Vol.} = 0.1\ m^3$

$\rho_{Air} g V_{ol.} = 0.1$

$= 1.2\ kg/m^3$

$\Sigma F_y = m_v a$ up is positive

$(\rho_{Air} g V_{ol.}) - m_v g = m_v a$

$m_v = \frac{1}{2} \rho_{air} V_{ol.}$

$\rho_{Air} g V_{ol.} - \frac{1}{2} \rho_{air} V_{ol.} g = \frac{1}{2} \rho_{air} V_{ol.} a$

$\frac{1}{2} g = \frac{1}{2} a$

$\boxed{a = g \approx 10\ m/s^2 \text{ up}}$

$V = V_0 + at = 5$

$V_0 = 0$, $a = 10\ m/s^2$

$\boxed{V = 50\ m/s}$

$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 = 25\ s^2$

$y_0 = 0$, $v_{0y} = 0$, $a_y = 10\ m/s^2$

$y = \frac{250}{2}\ m = \boxed{125\ m}$

9. Geosynchronous JLA Watchtower

$T = P = 1 \text{ day} = 86\,400 \text{ sec}$

$M_E = 5.97 \times 10^{24} \text{ kg}$

Kepler $\quad T = 2\pi r^{3/2} / \sqrt{G M_E}$

$G = 6.67 \times 10^{-11} \text{ N}(\text{m/kg})^2$

$$r = \frac{T}{2\pi} \sqrt{G M_E}^{3/2}$$

$$r = \left(\frac{86400\text{s}}{2\pi} \sqrt{6.67\times10^{-11} \times 6\times10^{24}} \right)^{2/3}$$

$\frac{86400}{2\pi} \approx 13800$

$6.67\times10^{-11} \times 6\times10^{24} = 39.8\times10^{13} = 1.99\times10^{7}$

$$r = \left(2.75 \times 10^{12} \right)^{2/3}$$

$$r = 4.27 \times 10^{6} \text{ m} = 42{,}700 \text{ km}$$

(26500 miles)

Note: often altitude is given above sea level, $R_E \approx 6400$ km.