PHYSICAL REVIEW D VOLUME 42, NUMBER 4 15 AUGUST 1990

Physical effects in wormholes and time machines

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(Received 9 February 1990)

Physical effects in a spacetime with a traversable wormhole are considered. It is shown that the interaction of a wormhole with the surrounding matter and with the external gravitational field almost inevitably transforms it into a time machine.

I. INTRODUCTION

One of the most intriguing predictions of general relativity is the possibility of the existence of topologically nontrivial spacetimes. The real three-dimensional space might in principle be multiply connected and there might exist wormholelike objects in it.[1,2] Recently interest in this problem increased because it was shown that a stable wormhole (if only it exists) can be transformed into a time machine.[3–6]

One can imagine a wormhole as a three-dimensional space with two spherical holes (mouths) in it. These holes are connected one with another by means of a handle. The length of this handle l does not depend on the distance L between the mouths in external space and in principle this length might be much smaller than L. The two-dimensional section of such a space is schematically shown in Fig. 1. The wormholes with the stationary or slowly-changing-in-time geometry of the handle are of particular interest. It is possible to enter into such a wormhole, to pass through the tunnel, and to exit into external space again (traversable wormholes). This property distinguishes wormholes from black holes. In Refs. 3 and 4 it was shown that for the existence of traversable wormholes the stress-energy tensor must violate the average weak energy condition.

The mouths of the wormhole may be moving one with respect to another in an external space without any notable changes of the internal geometry of the handle. In Refs. 4 and 5 it was shown that this relative motion of the mouths can be chosen in such a way that there arise closed timelike world lines. It happens already in a practically flat Minkowski spacetime. This result means that there arises a "time machine" which allows one to return in his own past. The reason why it happens is rather simple and can be explained as follows.[4,5] Suppose that one of the mouths (A) is at rest in an inertial frame while the other (B) which initially was at rest near A begins to move with a high speed and afterwards returns back to B. As a result of the Lorentz time contraction, the time interval ΔT_B between these two events (the beginning of the motion and its end) measured by the clock moving with B can be made much shorter than the time interval ΔT_A between the same events measured by the clock at rest near A. In other words, the clock which moved has slowed by $\Delta T_A - \Delta T_B$ relative to the standard inertial clock ("the twin paradox"). The short distance through the handle between A and B remains practically unchanged during the motion so that along this way both clocks all the time remain practically at rest near one another. Their relative time difference is determined only by noninertial effects connected with the acceleration of B and this difference for short handles can be made negligibly small. This means that an observer comparing the time of the clocks through the handle will see that their time is almost the same, while by comparing them in external space he will find out that their time difference is $\Delta T_A - \Delta T_B$. Consider now an observer who enters the handle through B at the moment when the clock at B shows some time T_0 (the event p_0). This observer goes out to the external space through A (the event p_0') when the clock near A shows approximately the same time T_0. But the time measured by clock A coincides with the standard inertial time while clock B after its motion was slowed down. This means that the observer after passing the handle will be transferred into the past (as seen by the clocks in the inertial frame) by the time interval $\Delta T_A - \Delta T_B$. If this time interval is larger than the time L/c needed for a causal signal to propagate from A to B in the external space then the event p_0' lies in the causal past of the event p_0. In other words, such a device can be

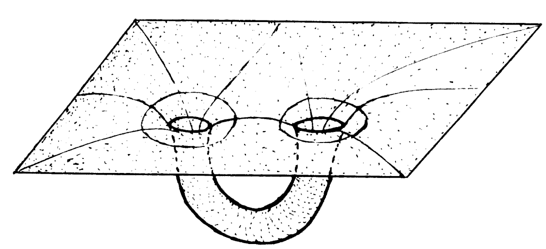

FIG. 1. A two-dimensional section of a static spacetime with a wormhole.

used as a time machine and in principle it allows one to return into his own past.

It seems that the physical laws do not forbid the existence of wormholes and time machines[3–6] but an assumption about their existence creates a lot of problems. The most important one is the problem of causality. This problem is analyzed in Ref. 6.

The main aim of this paper is to analyze some physical processes in spacetimes with wormholes. Namely, we consider a wormhole which is inserted in an external electromagnetic or gravitational field and we describe nontrivial effects which may happen in this system. In particular we show that almost any wormhole placed in an external gravitational field or interacting with external matter becomes a time machine.

This paper is organized as follows. In the next section we consider some simple models of static wormholes and use these models to analyze what happens when the wormhole is affected by an external static gravitational field. The general properties of static wormholes in an external gravitational field are considered in Sec. III. The electrodynamics of wormholes is discussed in Sec. IV. Section V contains some additional general remarks concerning the physical properties of spacetimes with wormholes.

In this paper we use the natural units $c=G=\hbar=1$ and the sign conventions of Ref. 7.

II. WORMHOLE GEOMETRY

We begin by considering some of the properties of a static spherically symmetric wormhole connecting two different asymptotically flat spaces. (For a general and more detailed discussion of wormholes, see Ref. 3). The embedding diagram for such a wormhole is shown in Fig. 2. The metric for this spacetime reads

$$ds^2=-\alpha^2 dt^2+dl^2+r^2 d\omega^2 , \tag{2.1}$$

where $d\omega^2=d\theta^2+\sin^2\theta\, d\varphi^2$ is a line element on a unit sphere. The proper radial distance l from the wormhole throat ranges from $-\infty$ to $+\infty$ and $l=0$ at the throat. The functions $\alpha(l)$ and $r(l)$ possess the following properties. The value of r decreases from $+\infty$ (for $l=-\infty$) to a minimum $r=b$ at the throat $l=0$ and increases to infinity (for $l=+\infty$). The spacetime (2.1) is static and the redshift function $\alpha(l)$ is connected with the norm of the Killing vector: $\alpha^2=-\xi^\mu\xi_\mu$ ($\xi^\mu\partial_\mu=\partial_t$). We assume that α is positive everywhere and hence there is no event horizon. Denote the asymptotic values of $\alpha(l)$ at $l=\pm\infty$ by $\alpha^0_\pm$, correspondingly. It should be stressed that we cannot require that $\alpha^0_+=\alpha^0_-$ and we shall see that, in general, $\alpha^0_+\neq\alpha^0_-$. Because this point is important for considerations we discuss it in more detail.

The metric (2.1) is a solution of the Einstein equations with a nonvanishing right-hand side $T_{\mu\nu}$. The stress-energy tensor $T_{\mu\nu}$ necessarily violates the averaged weak energy condition.[3,4] For our consideration it is instructive to consider at first the following simple model of a wormhole. Namely, we assume that matter which creates the gravitational field of a wormhole is localized in a narrow region $l\in(-\epsilon,\epsilon)$ near the throat (Fig. 3). Outside this region the spacetime geometry coincides with the Schwarzschild one and it is described by the metric (2.1) with

$$\alpha_\pm(l)=\alpha^0_\pm(1-2M_\pm/r_\pm)^{1/2} , \tag{2.2}$$

$$\frac{dr_\pm}{dl}=\pm(1-2M_\pm/r_\pm)^{1/2} . \tag{2.3}$$

The signs $+$ and $-$ are used for quantities in R_+ and R_- regions, correspondingly. The parameters M_+ and M_- are the masses of the wormhole as measured by distant observers in R_+ and R_- spaces, correspondingly. In the general case in order to specify the parameters $\alpha^0_\pm$ and $M_\pm$ one needs to know the solution of the Einstein equations in the region with matter. But in our case, where this region is narrow it is enough to know only some integrated characteristics of the matter distribution

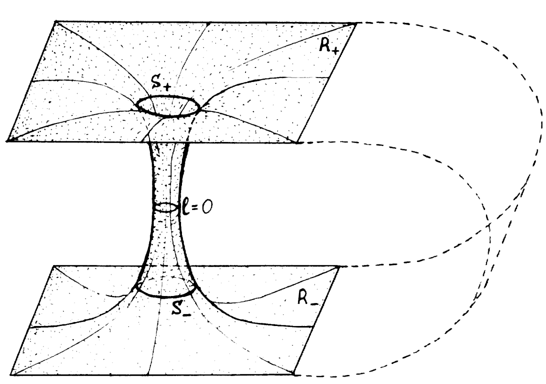

FIG. 2. The embedding diagram for a two-dimensional section t=const, $\theta=/2$ for a static spherically symmetric wormhole, connecting two different asymptotically flat spaces R_+ and R_-.

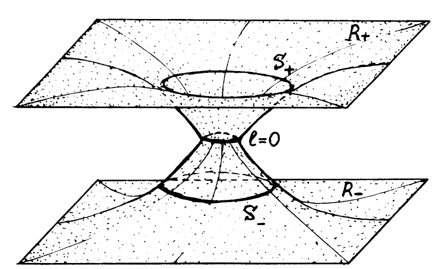

FIG. 3. The embedding diagram for a two-dimensional section t=const, $\theta=/2$ for a static spherically symmetric wormhole, connecting two different asymptotically flat spaces R_+ and R_- in the limiting case when the matter creating the gravitational field of the wormhole is located in a narrow region near its throat.

and one can use the method of massive thin shells.[8,9]

Denote

$$S_n^m = \lim_{\epsilon\to 0} \int_{-\epsilon}^{\epsilon} T_n^m dl \quad (n,m=0,2,3) , \tag{2.4}$$

$$r_+(0)=r_-(0)=b , \tag{2.5}$$

and

$$\varphi_\pm = M_\pm / b . \tag{2.6}$$

Then one has

$$\alpha_+^0(1-2\varphi_+)^{1/2}=\alpha_-^0(1-2\varphi_-)^{1/2} , \tag{2.7}$$

$$S_0^0=(4\pi b)^{-1}[(1-2\varphi_+)^{1/2}+(1-2\varphi_-)^{1/2}] , \tag{2.8a}$$

$$S_2^2=S_3^3=(8\pi b)^{-1}\left[\frac{1-\varphi_+}{(1-2\varphi_+)^{1/2}}+\frac{1-\varphi_-}{(1-2\varphi_-)^{1/2}}\right] . \tag{2.8b}$$

Other components of S_n^m vanish.

For given values of the mass density S_0^0 and pressure $S_2^2=S_3^3$ of the massive thin shell of radius b, Eq. (2.8) allows one to obtain the masses M_+ and M_-. Equation (2.7) shows that $\alpha_+^0=\alpha_-^0$ if and only if the masses M_+ and M_- are identical. This result has a very simple physical interpretation. The quantities $\varphi_\pm$ may be considered as gravitational potentials. Let us consider a photon propagating through the wormhole from R_+ to R_-. Its frequency becomes blueshifted when it is moving from R_+ infinity to the throat where the gravitational potential is less by the value φ_+. The redshift of the frequency of this photon during its further motion from the throat to R_- infinity is determined by the gravitational potential φ_-. If we denote by $\omega_\pm$ the frequencies of the photon at $R_\pm$ infinities then we have

$$\frac{\omega_-}{\omega_+}=\frac{\alpha_+^0}{\alpha_-^0} . \tag{2.9}$$

Now we show that even if initially the wormhole was "prepared" in such a way that $\alpha_+^0=\alpha_-^0$ this equality will be violated as soon as this wormhole is inserted into an external gravitational field or some matter is placed close to it. In order to make the consideration more concrete we assume that in space R_+, there is a spherically symmetric cloud of matter surrounding the mouth S_+. For any spherically symmetric motion of the cloud there is no gravitational radiation propagating through the throat to the R_- infinity and hence the mass M_- remains unchanged; i.e., it does not depend on the radius B of the cloud. Thus for fixed parameters of the matter distribution (2.8) in the throat of the wormhole the spacetime geometry in R_- and in the part $r_+<B$ of R_+ remains the same as earlier and is described by Eqs. (2.2) and (2.3). The geometry outside the cloud is again the Schwarzschild one with some new mass which we denote M_+^1. This mass depends on the wormhole parameters and the mass of the cloud. If the cloud is thin one can use again the massive thin shells method in order to obtain this dependence in explicit form. Namely, one has

$$\alpha_+^1(1-2\Phi_+^1)^{1/2}=\alpha_+^0(1-2\Phi_+)^{1/2} , \tag{2.10}$$

$$\tilde{S}_0^0=(4\pi B)^{-1}[(1-2\Phi_+^1)^{1/2}-(1-2\Phi_+)^{1/2}] , \tag{2.11a}$$

$$\tilde{S}_2^2=\tilde{S}_3^3=(8\pi B)^{-1}\left[\frac{1-\Phi_+^1}{(1-2\Phi_+^1)^{1/2}}-\frac{1-\Phi_+}{(1-2\Phi_+)^{1/2}}\right] , \tag{2.11b}$$

where

$$\Phi_+=\frac{M_+}{B}, \quad \Phi_+^1=\frac{M_+^1}{B} , \tag{2.12}$$

and $\tilde{S}_0^0$ and $\tilde{S}_2^2=\tilde{S}_3^3$ are the mass density and pressure of the shell. By using Eqs. (2.7) and (2.10) we can obtain

$$\frac{\alpha_+^1}{\alpha_-^0}=\left[\frac{(1-2\Phi_+)(1-2\varphi_-)}{(1-2\Phi_+^1)(1-2\varphi_+)}\right]^{1/2} . \tag{2.13}$$

This relation shows that for given wormhole parameters the relative redshift factor at R_+ infinity with respect to R_- infinity depends on the mass and position of the cloud surrounding the wormhole and the equality $\alpha_+^1=\alpha_-^0$ can be valid only for a very special configuration. This result is of quite a general nature. Even if the wormhole was originally "prepared" in such a way that $\alpha_+^0=\alpha_-^0$ this relation will be almost always violated as soon as the wormhole is inserted in an external gravitational field so that there arises a difference of the gravitational potentials between its two mouths.

Now consider a static wormhole which connects far distant regions in the same asymptotically flat spacetime. Such a spacetime can be obtained from the spacetime shown in Fig. 2 by identifying infinite almost flat regions of R_+ and R_- (for details see Ref. 3). In the general case for a wormhole connecting two different spaces $\alpha_+^0\neq\alpha_-^0$ and the spacetime which arises as the result of identificaton of R_+ and R_- infinities possesses a rather remarkable property: the work done by the gravitational field on any particle which propagates along any closed path passing through the wormhole is not equal to zero and hence the gravitational field is nonpotential. This property of spacetime can be formulated in a more geometrical way. The redshift function α coincides with the norm of the Killing vector. At any given point the norm of the Killing vector can be fixed arbitrarily but after this the Killing vector is unique defined in any simply connected region surrounding this point. In a multiply connected spacetime it is not so. Fix the norm of the Killing vector at some point and consider the changes of this vector along a path passing through the wormhole and returning back to the initial point. The initial and final Killing vectors have the same direction but their norms are different. This means that the Killing vector field is well-defined locally but it does not exist globally. In the next section we consider the general properties of such spacetimes.

III. STATIC SPACETIMES WITH WORMHOLES: GENERAL PROPERTIES

Consider a spacetime, i.e., a four-dimensional manifold M with a metric $g_{\mu\nu}$. Consider a region U of the spacetime M. The spacetime is called *stationary* in $U \subset M$ if there exists a timelike vector field ξ^μ such that

$$\mathcal{L}_\xi g_{\mu\nu} = 0 \ . \tag{3.1}$$

Here $\mathcal{L}_\xi$ is the Lie derivative along ξ^μ. The field ξ^μ is a Killing vector and Eq. (3.1) is identical to the usual Killing equation

$$\xi_{(\mu;\nu)} = 0 \ . \tag{3.2}$$

The spacetime is static in U if the Killing field obeys the equation

$$\xi_{[\mu;\nu}\xi_{\lambda]} = 0 \ . \tag{3.3}$$

It is easy to show that any Killing field satisfies the relation

$$\zeta_{\mu\nu;\beta} = R_{\mu\nu\beta\gamma}\xi^\gamma \ , \tag{3.4a}$$

where

$$\xi_{\mu;\nu} = \zeta_{\mu\nu}, \quad \zeta_{\mu\nu} = \zeta_{[\mu\nu]} \ . \tag{3.4b}$$

This equation shows that all higher covariant derivatives of ξ^μ can be expressed in terms of ξ^μ and its first derivatives.

Consider integral lines $x^\mu(t)$ of the Killing field ξ^μ:

$$\frac{dx^\mu}{dt} = \xi^\mu \ . \tag{3.5}$$

The parameter t is called the Killing time. It should be stressed that the norm of the Killing vector field is not fixed. We can change it:

$$\xi^\mu \to \xi'^\mu = \beta\xi^\mu, \quad \beta = \text{const} \ . \tag{3.6}$$

Under this transformation the Killing time is changed as follows:

$$t \to t' = \beta^{-1} t \ . \tag{3.7}$$

One may consider a test body which is moving along an integral lines (trajectory) of the Killing field. The four-velocity of this body is

$$u^\mu = \frac{\xi^\mu}{|\xi^2|^{1/2'}} \tag{3.8}$$

and its four-acceleration w^μ reads

$$w^\mu \equiv u^\nu \nabla_\nu u^\mu = \tfrac{1}{2}\nabla^\mu \ln|\xi^2| \ . \tag{3.9}$$

In the general case this acceleration does not vanish. This means that such a body must be affected by some external nongravitational force. One can attach a proper reference frame to the chosen body. The proper time in this (Killing) reference frame is

$$d\tau = |\xi^2|^{-1/2} dt \ . \tag{3.10}$$

(For more detailed discussion of the Killing frames, see Ref. 10.) It is evident that $u^\mu w_\mu = 0$ and

$$w_{[\mu;\nu]} = 0 \ . \tag{3.11}$$

The four-velocity u^μ and four-acceleration w^μ of the Killing observer are invariant under the rescaling transformation (3.6).

For given values of ξ^μ and $\zeta_{\mu\nu}$ at a fixed point Eqs. (3.4) unambiguously define ξ^μ in any simply connected region surrounding this point. But in the general case, as we have seen, there may be no global Killing field while locally the Killing field exists in the vicinity of any spacetime point. In order to be able to deal with this kind of situation it is useful to reformulate the definition of a static spacetime in terms of the quantities u^μ and w^μ which have a well-defined physical meaning, are invariant under scale transformations (3.6), and hence are uniquely defined.

It is possible to show that in any simply connected region of a spacetime the system of Eqs. (3.2) and (3.3) is equivalent to the following system of equations for vectors u^μ and w^μ:

$$u_\mu u^\mu = -1 \ , \tag{3.12}$$

$$u_{\mu;\nu} = -w_\mu u_\nu \ , \tag{3.13}$$

$$w_{[\mu;\nu]} = 0 \ . \tag{3.14}$$

We shall call "static" any (not necessarily simply connected) spacetime which admits global vector fields u^μ and w^μ obeying the conditions (3.12)–(3.14).

Now consider the properties of "static" spacetimes. Eq. (3.14) shows that the one-form $w \equiv w_\mu dx^\mu$ is closed,

$$dw = 0 \ , \tag{3.15}$$

and hence the integral of w over any closed path C^1,

$$I_{\{C^1\}}[w] \equiv \int_{C^1} w = \oint_{C^1} w_\mu dx^\mu \ , \tag{3.16}$$

depends only upon the homology class of the path. The value of this integral is called the period of w on $\{C^1\}$, the homology class of C^1 (Ref. 2). In a simply connected space any closed path is homological to a point and $I_{\{C^1\}}[w] = 0$. In other words, the form w is exact:

$$w = d\varphi \ . \tag{3.17}$$

It is easy to show that

$$\varphi = \tfrac{1}{2}\ln|\xi^2| + \varphi_0, \quad \varphi_0 = \text{const} \ . \tag{3.18}$$

In the general case of a multiply connected "static" spacetime the vanishing of the periods $I_{\{C^1\}}[w]$ for all closed paths is the necessary and sufficient condition of exactness of the closed form w and hence of the global existence of the Killing field ξ^μ:

$$\xi^\mu = e^\varphi u^\mu \ . \tag{3.19}$$

The possibility of the existence of a time machine in a static spacetime which was described in the Introduction is a quite general property of multiply connected spaces. In order to show this we consider at first the problem of clock synchronization in a static multiply connected spacetime with the potential gravitational field. Using Eqs. (3.13), (3.18), and (3.19) one can show that the vector $\eta^{\mu}=e^{-\varphi}u^{\mu}\equiv\xi^{\mu}/|\xi^2|$ obeys the relation

$$\eta_{[\mu,\nu]}=0 \ . \tag{3.20}$$

In a simply connected region this equation implies the existence of a time function t such that

$$\eta_{\mu}=\nabla_{\mu}t \ . \tag{3.21}$$

The surface $t=$const is the set of events which are simultaneous in the reference frame of Killing observers. The period

$$I_{\{C^1\}}[\eta]\equiv\int_{C^1}\eta=\oint_{C^1}\eta_{\mu}dx^{\mu} \tag{3.22}$$

of the one-form η vanishes for any closed path C^1 lying in a simply connected region. This result means that the result of the synchronization procedure for two clocks placed at any two points in this region does not depend on the particular choice of the path connecting these points. In a multiply connected spacetime the period $I_{\{C^1\}}[\eta]$ depends on the homology class of the closed path C^1 and in the general case it does not vanish. In a static spacetime with one wormhole the homology class of closed paths can be specified by an integer winding number n which defines how many times the chosen closed path passes through the throat of the wormhole in the direction from the S_+ mouth to the S_- mouth. For the opposite direction of a path the number n is negative. Denote Δ_1 as the value of the period (3.22) for $n=1$. If $\Delta_1\neq 0$ then the form η does not allow globally the representation (3.21) and it is impossible to synchronize the clocks along any closed path passing through the wormhole. The quantity Δ_1 as we shall show later gives the gap between the initial value of the Killing time and the value of this time at the same point after the synchronization along the path passing through the wormhole. The quantity Δ_1 is well defined for a static spacetime. Suppose that the mouths of the wormhole are moving in the external space during some time but before this period and after it the spacetime is static. A simple generalization of the arguments presented in the Introduction allows one to conclude that the value of Δ_1 after the motion may differ from its initial value. A time machine is created if, after the motion, $\Delta_1>L$, where L is the distance between the mouths.

Consider now a general situation where the spacetime of a wormhole is "static" and the gravitational field is nonpotential. We shall show that in this case the time machine is created even without any motion of the wormhole's mouths. In a "static" spacetime with a wormhole the period $I_{\{C^1\}}[w]$ depends on the winding number of a path C^1. For a winding number n the period $I_n[w]$ of w is $I_n[w]=nI_1[w]$. The period $I_1[w]$ has a simple physical meaning. In order to discuss it we need to make some additional remarks.

For a description of the properties of a "static" spacetime it is convenient to use the three-dimensional formalism developed by Geroch.[11] Let S denote the collection of all the trajectories of Killing observers u^{μ}. That is, an element of S is a curve γ in M which is everywhere tangent to u^{μ}. For each point of M one can find the trajectory of u^{μ} which passes through this point so that there exists the natural projection mapping ψ: $M\to S$. One may consider S as a three-dimensional manifold. For a spacetime with a wormhole the space S is multiply connected. The three-dimensional metric h on S is connected with the four-dimensional metric on M as follows:

$$h_{\mu\nu}=g_{\mu\nu}+u_{\mu}u_{\nu} \ . \tag{3.23}$$

Consider now the problem of clock synchronization in S along a curve Γ. Let U be a simply connected region in S and denote by U_{ψ} a simply connected four-dimensional region $\psi^{-1}U$ in M. Equation (3.13) allows one to show that the one-form $u=u_{\mu}dx^{\mu}$ in M obeys the condition

$$u\wedge du=0 \tag{3.24}$$

and hence in accordance with the Frobenius theorem this form in U_{ψ} can be written as

$$u=\lambda(x)dt \ . \tag{3.25}$$

Denote by Σ_t a three-dimensional spacelike surface defined by the equation $t=$const. The trajectories of u^{μ} are orthogonal to Σ_t and for a fixed value of t the surface Σ_t is formed by events which are simultaneous with one another in the reference frame of the u observer. The surface Σ_t crosses any u trajectory in U_{ψ} only once.

Any point p_0 in U_{ψ} defines in a unique way the trajectory γ_0 and the surface Σ_0 to which it belongs. Denote by the same letter γ_0 the point of S determined by the trajectory γ_0 and consider a path Γ in S which begins at this point. This path can be unambiguously lifted to U_{ψ} if we require that the corresponding lifted path $\sigma_{\Gamma}[p_0]$ lying on Σ_0. The events on $\sigma_{\Gamma}[p_0]$ are simultaneous with p_0. This procedure provides the synchronization of clocks along any curve in U. If one considers two different curves in U which connect the initial point γ_0 with the same final point γ_1 the synchronization procedure along both curves will give identical results in γ_1. The parameter t defined by Eq. (3.25) may be considered as the "universal" time in U_{ψ}. It is the existence of this "universal" time which makes possible a noncontradictory synchronization along any closed curve in U.

The situation is quite different if we consider clock synchronization in a multiply connected space S. Consider a closed path $\Gamma{:}\gamma(\lambda)$ in S which passes through the throat of a wormhole. Denote by Γ_{ψ} a two-dimensional surface in M generated by a one-dimensional family Γ of u trajectories and denote by γ_0 the $\gamma(0)$ trajectory which corresponds to $\lambda=0$ (see Fig. 4). For a given event p_0 on γ_0 we may define the set $\sigma_{\Gamma}[p_0]$ of events which are simultaneous with p_0 along any curve Γ beginning at γ_0. This path $\sigma_{\Gamma}[p_0]$ is the intersection of a surface Σ_0 passing through p_0 with the surface Γ_{ψ}. For $\lambda=\lambda_0$ this path crosses γ_0 again at some point p_0'. In general case p_0' does not coincide with p_0. For this reason the global synchronization of clocks in such a spacetime is impossible. (The analogous situation is well known for stationary but nonstatic spacetimes where it is impossible to synchronize the clocks already in a simply connected region.)

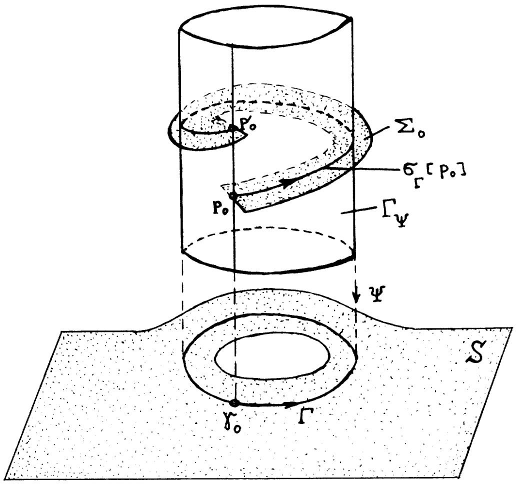

FIG. 4. The clock synchronization in a multiply connected "static" spacetime.

Denote by $\Delta_\Gamma \tau[p_0]$ the difference between the proper times for the events p'_0 and p_0 on γ_0. It can be shown that for a given point p_0 the "time gap" $\Delta_1 \tau[p_0]$ depends only on the homology class of the path Γ in S which passes through γ_0. For the "static" spacetime with one wormhole the value of $\Delta_\Gamma \tau[p_0]$ depends on the winding number n of the path and we denote it $\Delta_n \tau[p_0]$. For a given u trajectory $\Delta_n \tau[p_0]$ depends on the proper time τ.

Now we prove that the quantity $d\Delta_n \tau[p_0]/d\tau$ does not depend on the particular choice of the u trajectory and

$$\frac{d\Delta_1 \tau}{d\tau} = e^{I_1[w]} - 1 \ . \tag{3.26}$$

In order to prove this we consider two points p_0 and q_0 on γ_0 separated by the proper time interval $\delta\tau_0$. Let Γ be a closed curve in S which passes through γ_0 and goes through the handle of the wormhole. Consider a finite part of this curve and let p_1 and q_1 be the points on the trajectory γ_1 which are simultaneous along Γ with p_0 and q_0 correspondingly. In the vicinity U_ψ of the $\gamma_0\gamma_1$ *path* in S one can introduce the "universal" Killing time t. Denote by δt_i and $\delta\tau_i$ the interval of the Killing and proper time between p_i and q_i along γ_i, correspondingly. It is evident that $\delta t_0 = \delta t_1$ and hence

$$\frac{\delta\tau_0}{\delta\tau_1} = \frac{|\xi^2(p_0)|^{1/2}}{|\xi^2(p_1)|^{1/2}} \ . \tag{3.27}$$

This relation does not depend on the particular choice of the norm of the Killing vector in U_ψ. In this simply connected region Eq. (3.27) can be rewritten in the form

$$\ln \frac{\delta\tau_1}{\delta\tau_0} = \int_{\gamma_0}^{\gamma_1} w \ . \tag{3.28}$$

If the end point $\gamma(\lambda_0)$ coincides with the initial one γ_0 the integral on the right-hand side of (3.28) gives $I_1[w]$. By using this relation one gets

$$\delta\tau'_0 - \delta\tau_0 = \delta\tau_0[e^{I_1[w]} - 1] \ , \tag{3.29}$$

where $\delta\tau'_0$ is the proper time distance between the points p'_0 and q'_0 on γ_0 (see Fig. 5). If $\Delta_1\tau[p_0]$ is the "time gap" for the synchronization along a closed path with a winding number $n=1$ beginning at the event p_0 and $\Delta_1\tau[q_0]$ is the analogous "time gap" for the events q_0 then

$$\Delta_1\tau[q_0] - \Delta_1\tau[p_0] = \delta\tau_0[e^{I_1[w]} - 1] \ . \tag{3.30}$$

This relation shows that the "time gap" changes with the proper time along γ_0 at a rate given by Eq. (3.26). The independence of $d\Delta_n\tau/d\tau$ on the particular choice of the u trajectory is just a consequence of the invariance of the period $I_n[w]$.

There are two physically important results which follow from the above considerations. Consider a traversable wormhole in an external gravitational field. For example, suppose that one of its mouths is moved close to the surface of a neutron star and it is held there at rest by some external force while the other mouth remains far from the star. For such a system the period $I_1[w]$ does not vanish and the "time gap" for the clock synchronization in the external space with respect to synchronization through the handle grows with time

$$\Delta_1\tau = [e^{I_1[w]} - 1](\tau - \tau_0) \ , \tag{3.31}$$

where τ_0 is the time when one of the mouths was inserted into the gravitational field. When $\Delta_1\tau$ becomes larger than the time T of the light propagation in the external space between the mouths, the wormhole becomes a time machine. In a weak gravitational field $I_1[w] \simeq \Delta\varphi$, where $\Delta\varphi$ is the difference of the gravitational potentials between the mouths and the condition of the time machine creation is

$$\tau - \tau_0 > \frac{T}{\Delta\varphi} \ . \tag{3.32}$$

Equation (3.28) also shows that the redshift factor for a

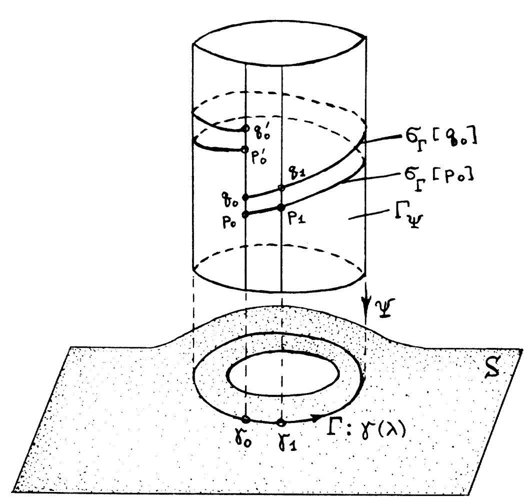

FIG. 5. Illustration to the proof of the Eq. (3.26).

photon which passes though the wormhole is

$$\alpha = e^{I_1[w]} . \tag{3.33}$$

This means that photons (as well as any other particles) which pass through the wormhole can be used to extract energy from it. For the wormhole in a weak gravitational field the time needed for a photon to go through the wormhole and return back is much shorter than the time (3.32) for time-machine creation so that during a rather long time one can use the wormhole as the energy generator before a nontrivial causal structure will develop. What is the origin of the energy which can be obtained by this process? In order to answer this question it is instructive to analyze the analogous problem in electrodynamics.

IV. WORMHOLES ELECTRODYNAMICS

For simplicity we consider at first the case where $I_1[w]=0$; i.e., the gravitational field of the wormhole is potential. We write the four-dimensional Maxwell equations in the form

$$d{}^*f = 4\pi {}^*j , \tag{4.1}$$

$$df = 0 . \tag{4.2}$$

Here $f = f_{\mu\nu} dx^\mu \wedge dx^\nu$ is the electromagnetic two-form. These equations can be used to prove some important integral laws in a multiply connected spacetime.[2] The electric $4\pi q[C_i^2]$ and magnetic $4\pi p[C_i^2]$ field fluxes through the two-dimensional surface C_i^2 are

$$4\pi q[C_i^2] = \int_{C_i^2} {}^*f , \tag{4.3}$$

$$4\pi p[C_i^2] = \int_{C_i^2} f . \tag{4.4}$$

For any two homological surfaces C_1^2 and C_2^2: $C_2^2 - C_1^2 = \partial C^3$, one has

$$q[C_2^2] - q[C_1^2] = \int_{C^3} {}^*j , \tag{4.5}$$

$$p[C_2^2] - p[C_1^2] = 0 . \tag{4.6}$$

Equation (4.6) shows that the magnetic flux through C^2 depends only on the homology class $\{C^2\}$ of a closed two-dimensional surface C^2 and does not change in time. If we choose a part of the surface $t=$const as C^3 then Eq. (4.5) shows that the difference between the electric field fluxes through two homological closed two-surfaces is $4\pi Q$, where Q is the electric charge in the three-volume C^3 bounded by these surfaces. On the other hand, if C_2^2 is just the "late time version" of C_1^2 (and hence it is represented by the same two-surface in S) then Eq. (4.6) shows that the change of the electric field flux through this surface is determined by the total charge which enters through this surface.

For our particular case of a static wormhole the latter result means that the electric field flux through the handle cannot be changed until charged particles pass through it. Any motion of electric charges in the external space does not change the value of the electric and magnetic field fluxes through the wormhole.

Consider now a static electric field created by electric charges located outside a wormhole. We consider effects connected with the electric field, so we put the magnetic field equal to zero. It is convenient to use the three-dimensional formalism[2] (see also Ref. 10). Let $x^i (i=1,2,3)$ be the coordinates in S. Denote by $e = F_{i\mu}\xi^\mu dx^i$ a three-dimensional one-form of the electric field strength. It obeys the equations

$$\delta e = 4\pi\rho , \tag{4.7}$$

$$de = 0 , \tag{4.8}$$

where $\delta = (-1)^p {}^*d{}^*$ is an operator of codifferential (or divergence) of a p-form and $\rho = j_\mu \xi^\mu$ is the charge density. The electromagnetic energy of a system can be written in the form

$$E = \frac{1}{8\pi}\int e \wedge {}^*e \equiv \frac{1}{8\pi}\int d^3v\, e_i e^i . \tag{4.9}$$

The Kodaira theorem[12,13] allows one to show that any finite-energy solution of Eqs. (4.7) and (4.8) can be uniquely decomposed into a sum of two-forms,

$$e = e_1 + e_2 , \tag{4.10}$$

such that $e_1 = d\varphi$ is an exact form while the other one e_2 is harmonic: $\delta e_2 = de_2 = 0$. The forms e_1 and e_2 satisfy the orthogonality condition

$$\int e_1 \wedge {}^*e_2 = 0 . \tag{4.11}$$

In other words, the field e_1 is potential; i.e., it is the gradient of a scalar potential which is uniquely defined for a given charge distribution as a solution of the Poisson equation on S:

$$\Delta\varphi = 4\pi\rho . \tag{4.12}$$

The field e_2 is the nonpotential part of the electric field. In the absence of charges (when $e_1=0$) the field e_2 is uniquely defined by its flux through the wormhole's throat. The field lines of e_2 are closed. They enter the wormhole through one of the mouths (S_-) and go out through the other (S_+). In the external space this neutral wormhole with a trapped electric field e_2 looks like a dipole with positive charge being placed at S_+ and negative charge being placed at S_- (Ref. 2).

Consider now a neutral wormhole with $e_2=0$ and suppose that the electric charge is inserted from infinity and placed in the vicinity of one of its mouths. This charge creates an electric field e whose potential part e_1 has a nonvanishing potential difference between the mouths in the external space and hence the same potential difference must be created for it along the handle. Thus there arises some e_1-field flux through the handle. But the complete electric flux of the field e through the handle remains unchanged and must vanish. Hence during this process there must also arise a harmonic component e_2 in order to compensate the flux of e_1. This situation is similar to the one we have discussed in the gravitational case: the insertion of a wormhole into an external field generates the nonpotential component of the field.

A charged particle moving in the nonpotential electric field acquires energy. The work done by the field on the charged field particle is compensated by the energy decrease of the electric field. In order to show this consider a charged particle with charge Δq which initially is at rest near the wormhole and then passes through the wormhole and returns back to the initial position. We suppose that the energy gained is used to do some work and the final configuration of charges coincides with initial one. Thus the component e_1 of the field remains unchanged but the electric flux through the handle is changed by the value $4\pi\Delta q$. This implies a change of e_2. Using Eqs. (4.9) and (4.13) one can write the energy in the form

$$E=\frac{1}{8\pi}\int d^3v[e_{1_i}e_1^i+e_{2_i}e_2^i] \ . \tag{4.13}$$

It is possible to show that the decrease of the electric energy E due to the change of e_2 coincides with the work done by the field on the charged particle during its motion along the closed path. The process of the energy extraction can be continued until the field component e_2 vanishes. For a given charge distribution the configuration with the potential electric field possesses the lowest possible energy and hence it is equilibrium state. There is a simple explanation why this happens. Consider a cyclic motion of a charge Δq which passes through a wormhole with a potential electric field. The work done by the field on the charge along the closed path vanishes and hence there is no contribution to the energy change proportional to Δq. On the other hand, the second order in the Δq contribution to the energy change is positive. This happens because as the test particle passes the handle and leaves it through the mouth S_+ the effective electric charge of this mouth becomes less by the value Δq. In other words, the charged particle induces an additional negative charge $-\Delta q$ on S_+ and there arises an additional force of attraction of the particle to this induced charge.

The situation is quite different for the gravitational interaction. The gravitational field in the spacetime of a wormhole is potential for the equilibrium configuration. But this equilibrium corresponds to a maximum of the energy and it is unstable. Consider a motion of a massive test particle through the wormhole. After the massive particle leaves the wormhole through the mouth S_+ the mass of S_+ decreases and its attraction also decreases. If initially the masses of both mouths were equal and the gravitational field was potential then after the passage of a massive particle through the wormhole the masses of both mouths become different, the field becomes nonpotential, and the energy of the wormhole decreases. These arguments show that if there is matter outside the wormhole then the wormhole is unstable with respect to the processes which transform it into a time machine.

V. CONCLUDING REMARKS

In conclusion we make some additional remarks concerning the interesting possibilities which may arise if wormholes exist. Suppose that one of the mouths (say, S_-) of a traversable wormhole is placed near a black-hole horizon and is held there at rest by some external force, while the other mouth S_+ remains at a great distance from the black hole. Near the event horizon there exists a thermal atmosphere so that an observer at rest will see a thermal gas of Boulware quanta with the temperature $T_d \approx 2\pi/d$ where d is the distance from the horizon (see, e.g., Refs. 10, 14, and 15). Some of these thermal quanta can freely propagate through the wormhole and as a result the mouth S_+ becomes the source of thermal (with the temperature T_d) radiation. This process may be considered as a realization of the general idea of "mining" a blackhole proposed by Unruh and Wald.[16]

Another interesting possibility arises if the mouth S_- loses its support and begins to fall down into the black hole. In this case an observer at rest far from the black hole can use the wormhole to "study" the black hole's interior. This wormhole device in principle could be used to "save" an observer who had fallen down into a black hole earlier. At first sight this possibility to get information from the interior of a black hole contradicts the definition of a black hole. It is not so. The point is that the gravitational radius is only an apparent horizon. If the weak energy condition is satisfied then the apparent horizon either coincides with the event horizon or is hidden inside it and hence there exists a black hole (see, e.g., Refs. 17 and 18). On the other hand, a traversable wormhole can exist only if this energy condition is violated. Hence in the presence of a wormhole the event horizon may be located inside the apparent horizon (inside the gravitational radius) or simply be absent.

These remarks show that many interesting questions arise when one considers the interaction of a wormhole with a black hole. Nevertheless it should be emphasized once again that all these questions may have physical meaning only provided wormholes exist and a time machine is stable. The problem of the classical and quantum stability of the Cauchy horizon which exists inside any time machine as well as the problem of the consistency of systems with self-interaction in the presence of a time machine are still the main unsolved problems.

ACKNOWLEDGMENTS

The authors are grateful to Kip Thorne, Bob Wald, and Ulvi Yurtsever for helpful discussions and valuable remarks. One of the author's (V.P.F.) would like to thank Larry McLerran and the Theoretical Physics Institute at the University of Minnesota for hospitality during the preparation of this paper.

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