ASTR 5460, Fall 2011: Midterm

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Directions: This is a closed book exam, to be taken at home in up to two hours. You may use calculators. Material covers Ryden chapters 1-6. It is due in class Thursday, Oct. 27.

You may find the following two forms of the Friedmann equation of use answering problems:

$H(t)^2 = \frac{8\pi G}{3c^2}\epsilon(t) - \frac{\kappa c^2}{R_0^2 a(t)^2}$

$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$

Note that the energy density $\epsilon(t)$ includes contributes from matter, radiation, and the cosmological constant. Also note that H(t) is defined as $(\dot{a}/a)$.

Additional equations are the fluid equation and the equation of state:

$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$

and

$P = \omega\epsilon.$

Finally, you may also want to recall the Robertson-Walker metric:

$ds^2 = -c^2dt^2 + a(t)^2[dr^2 + S_k(r)^2 d\Omega^2]$ where $S_k(r) = r$ for $\kappa = 0$, $S_k(r) = R_0 sin(r/R_0)$ for $\kappa = 1$, and $S_k(r) = R_0 sinh(r/R_0)$ for $\kappa = -1$.

1. A long, long time ago in a galaxy far, far away...perhaps even in a different universe...the alien astronomer Edwoonoo Hoobooloo collects measurements of other galaxies. Velocities below are line of sight from Doppler effect, with positive numbers indicating redshifts. Distances come from studying variable stars with period-luminosity relationships calibrated from closer stars.

Galaxy	Distance (Mpc)	Velocity (km/s)
1	1	-40
2	1.5	170
3	2.2	200
4	2.5	300
5	2.7	250
6	3.1	310
7	3.3	390
8	3.7	410
9	4.0	370
10	5.5	580

Uncertainties on distance are approximately 10 percent. Uncertainties for velocity are negligible, but have only been corrected for motions of the planet and home star.

Assume a matter-dominated universe (ignore energy-density contributions from radiation and cosmological constant). **If the geometry of this universe is flat, how old is it? Show your work. Discuss uncertainties, both formal and systematic uncertainties, at least qualitatively.**

2. For each case below, how does the scale factor, a, evolve with time, and what is the age (in terms of H_0) in a universe that is:

a) Empty (also state what solutions are viable)
b) Filled only with radiation (for a spatially flat universe)
c) Filled only with matter (for a spatially flat universe)
d) Empty, flat, but with a non-zero cosmological constant

Be sure to show all your work and state what assumptions are made in each case.

3. Short answers. Please define, quantitatively when possible, and with plots when appropriate:

a. The Hubble Law.
b. The difference between the Hubble Constant and the Hubble Parameter.
c. The Hubble Distance.
d. The Hubble Time.
e. The proper distance.
f. Co-moving Volume.
g. The equivalence principle.
h. The null geodesic.
i. Olber's Paradox.
j. The scale factor.

4. Please provide a summary of what Clowe et al. (2006) discovered studying the Bullet Cluster. Say what it was about, the methods used, the results, and its historical context. The more details the better, and the more content-rich word count the better. Aim for 1-2 pages.