

## ASTR 5460 Final, December 2011

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Instructions: Closed book, closed note. Calculators are okay. Two hours maximum (one may be enough), take home, on your honor. Please turn this in to me or my mailbox no later than **5pm, Friday, Dec. 9.**

You may find the following forms of the Friedmann equation of use:

$$H(t)^2 = \frac{8\pi G}{3c^2}\epsilon(t) - \frac{\kappa c^2}{R_0^2 a(t)^2}$$

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

Note that the energy density  $\epsilon(t)$  includes contributions from matter, radiation, and the cosmological constant. Also note that the Hubble parameter  $H(t)$  is defined as  $(\dot{a}/a)$ . Additional equations available to are the fluid equation, the equation of state, and the acceleration equation:

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$$

$$P = \omega\epsilon$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P)$$

Finally, you may also want to recall the Robertson-Walker metric:

$ds^2 = -c^2 dt^2 + a(t)^2 [dr^2 + S_k(r)^2 d\Omega^2]$  where  $S_k(r) = r$  for  $\kappa = 0$ ,  $S_k(r) = R_0 \sin(r/R_0)$  for  $\kappa = 1$ , and  $S_k(r) = R_0 \sinh(r/R_0)$  for  $\kappa = -1$ .

1. A pair of binary quasars are observed at a redshift of  $z=0.6$ , with an angular separation of 1 arcsecond. They both have an apparent magnitude of  $m=17$ . Assuming a flat universe, with matter only, and a Hubble constant of  $72 \text{ km/s/Mpc}$ , please determine two things: their angular separation in kpc, and their absolute magnitude  $M$  (which should be the same for both). Note that  $1 \text{ parsec} = 3.08 \times 10^{16} \text{ meters}$ . Apparent and absolute magnitudes are equal at a distance of 10 parsecs. The concepts of luminosity distance and angular diameter distance may be useful.

2. Short answers:

a. Please explain what is the Jean's Length and how, very qualitatively and generally speaking, we need to modify the traditional physics for the case of large scale structure on cosmological scales.

b. What is the temperature of the microwave background radiation seen today? How big are the fluctuations (ignoring the dipole and galaxy)? What conditions must be satisfied for radiation and matter to decouple?

c. Please explain qualitatively why big bang nucleosynthesis produces mostly H and He, with only traces of other elements.

3. Estimate the Hubble parameter at the epoch of recombination. Please state all assumptions you make to answer this question and express your answer in  $\text{km/s/Mpc}$ .

4. Please explain how Adam Reiss et al. in 1998 determined that the expansion of the universe was accelerating. Be as detailed as possible from memory, and provide numbers when possible.