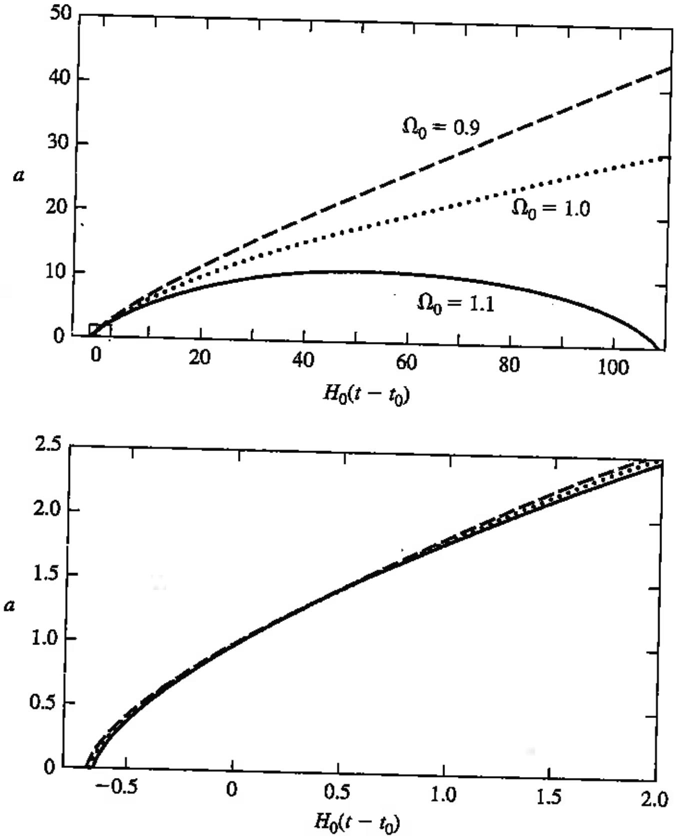

FIGURE 6.1 The scale factor as a function of time for universes containing only matter. The dotted line is $a(t)$ for a universe with $\Omega_0 = 1$ (flat); the dashed line is $a(t)$ for a universe with $\Omega_0 = 0.9$ (negatively curved); the solid line is $a(t)$ for a universe with $\Omega_0 = 1.1$ (positively curved). The bottom panel is a blow-up of the small rectangle near the lower left corner of the upper panel.

Crunch at $\theta = 2\pi$ is

$$t_{\text{crunch}} = \frac{\pi}{H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}. \tag{6.19}$$

A plot of a versus t in the case $\Omega_0 = 1.1$ is shown as the solid line in Figure 6.1. The $a \propto t^{2/3}$ behavior of an $\Omega_0 = 1$ universe is shown as the dotted line. The solution of equation (6.16) for the case $\Omega_0 < 1$ can be written in parametric form as

$$a(\eta) = \frac{1}{2} \frac{\Omega_0}{1 - \Omega_0} (\cosh \eta - 1) \tag{6.20}$$

The first term on the right-hand side of equation (6.23) represents the contribution of matter, and is always positive. The second term represents the contribution of a cosmological constant; it is positive if $\Omega_{m,0} < 1$, implying $\Omega_{\Lambda,0} > 0$, and negative if $\Omega_{m,0} > 1$, implying $\Omega_{\Lambda,0} < 0$. Thus, a flat universe with $\Omega_{\Lambda,0} > 0$ will continue to expand forever if it is expanding at $t = t_0$; this is another example of a Big Chill universe. In a universe with $\Omega_{\Lambda,0} < 0$, however, the negative cosmological constant provides an *attractive* force, not the repulsive force of a positive cosmological constant. A flat universe with $\Omega_{\Lambda,0} < 0$ will cease to expand at a maximum scale factor

$$a_{\max} = \left(\frac{\Omega_{m,0}}{\Omega_{m,0} - 1} \right)^{1/3}, \tag{6.24}$$

and will collapse back down to $a = 0$ at a cosmic time

$$t_{\text{crunch}} = \frac{2\pi}{3H_0} \frac{1}{\sqrt{\Omega_{m,0} - 1}}. \tag{6.25}$$

For a given value of H_0, the larger the value of $\Omega_{m,0}$, the shorter the lifetime of the universe. For a flat, $\Omega_{\Lambda,0} < 0$ universe, the Friedmann equation (6.23) can be integrated to yield the analytic solution

$$H_0 t = \frac{2}{3\sqrt{\Omega_{m,0} - 1}} \sin^{-1}\left[\left(\frac{a}{a_{\max}} \right)^{3/2} \right]. \tag{6.26}$$

A plot of a versus t in the case $\Omega_{m,0} = 1.1$, $\Omega_{\Lambda,0} = -0.1$ is shown as the solid line in Figure 6.2. The $a \propto t^{2/3}$ behavior of a $\Omega_{m,0} = 1$, $\Omega_{\Lambda,0} = 0$ universe is

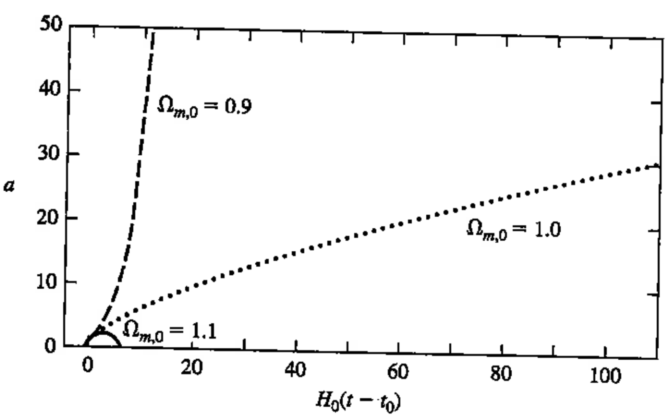

FIGURE 6.2 The scale factor as a function of time for flat universes containing both matter and a cosmological constant. The dotted line is $a(t)$ for a universe with $\Omega_{m,0} = 1$, $\Omega_{\Lambda,0} = 0$. The solid line is $a(t)$ for a universe with $\Omega_{m,0} = 1.1$, $\Omega_{\Lambda,0} = -0.1$. The dashed line is $a(t)$ for a universe with $\Omega_{m,0} = 0.9$, $\Omega_{\Lambda,0} = 0.1$.

Thus, it is possible to have a universe that expands outward at late times, but never had an initial Big Bang, with $a = 0$ at $t = 0$. Another possibility, if the values of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ are chosen just right, is a "loitering" universe.[5] Such a universe starts in a matter-dominated state, expanding outward with $a \propto t^{2/3}$. Then, however, it enters a stage (called the loitering stage) in which a is very nearly constant for a long period of time. During this time it is almost—but not quite—Einstein's static universe. After the loitering stage, the cosmological constant takes over, and the universe starts to expand exponentially.

Figure 6.3 shows the general behavior of the scale factor $a(t)$ as a function of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$. In the region labeled "Big Crunch," the universe starts with $a = 0$ at $t = 0$, reaches a maximum scale factor $a_{\max}$, then recollapses to $a = 0$ at a finite time $t = t_{\text{crunch}}$. Note that Big Crunch universes can be positively curved,

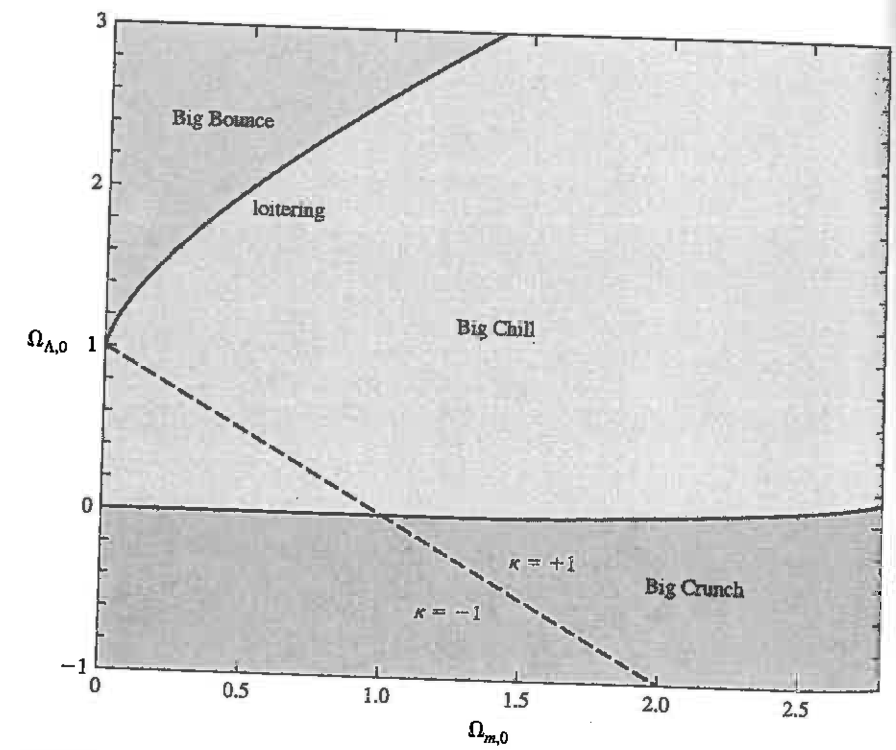

FIGURE 6.3 The curvature and type of expansion for universes containing both matter and a cosmological constant. The dashed line indicates $\kappa = 0$; models lying above this line have $\kappa = +1$, and those lying below have $\kappa = -1$. Also shown are the regions where the universe has a "Big Chill" expansion ($a \to \infty$ as $t \to \infty$), a "Big Crunch" recollapse ($a \to 0$ as $t \to t_{\text{crunch}}$), a loitering phase ($a \sim$ const for an extended period), or a "Big Bounce" ($a = a_{\min} > 0$ at $t = t_{\text{bounce}}$).

[5] A loitering universe is sometimes referred to as a *Lemaître universe*.

negatively curved, or flat. In the region labeled "Big Chill," the universe starts with $a = 0$ at $t = 0$, then expands outward forever, with $a \to \infty$ as $t \to \infty$. Like Big Crunch universes, Big Chill universes can have any sign for their curvature. In the region labeled "Big Bounce," the universe starts in a contracting state, reaches a minimum scale factor $a = a_{\rm min} > 0$ at some time $t_{\rm bounce}$, then expands outward forever, with $a \to \infty$ as $t \to \infty$. Universes that fall just below the dividing line between Big Bounce universes and Big Chill universes are loitering universes. The closer such a universe lies to the Big Bounce–Big Chill dividing line in Figure 6.3, the longer its loitering stage lasts.

To illustrate the different types of expansion and contraction possible, Figure 6.4 shows $a(t)$ for a set of four model universes. Each of these universes has the same current density parameter for matter: $\Omega_{m,0} = 0.3$, measured at $t = t_0$ and $a = 1$. These universes cannot be distinguished from each other by measuring their current matter density and Hubble constant. Nevertheless, thanks to their different values for the cosmological constant, they have very different pasts and very different futures. The dashed line in Figure 6.4 shows the scale factor for a universe with $\Omega_{\Lambda,0} = -0.3$; this universe has negative curvature, and is destined to end in a Big Crunch. The dotted line shows $a(t)$ for a universe with $\Omega_{\Lambda,0} = 0.7$; this universe is spatially flat, and is destined to end in an exponentially expanding Big Chill. The dot-dash line shows the scale factor for a universe with $\Omega_{\Lambda,0} = 1.7134$; this is a positively curved loitering universe, which spends

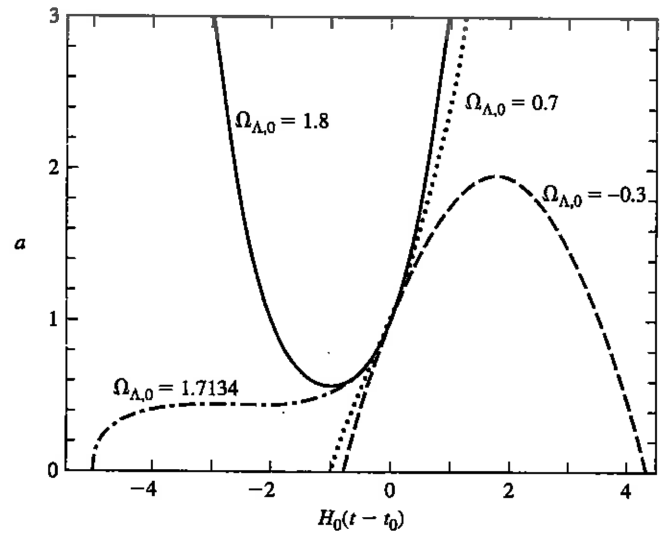

FIGURE 6.4 The scale factor a as a function of t in four different universes, each with $\Omega_{m,0} = 0.3$. The dashed line shows a "Big Crunch" universe ($\Omega_{\Lambda,0} = -0.3$, $\kappa = -1$). The dotted line shows a "Big Chill" universe ($\Omega_{\Lambda,0} = 0.7$, $\kappa = 0$). The dot-dash line shows a loitering universe ($\Omega_{\Lambda,0} = 1.7134$, $\kappa = +1$). The solid line shows a "Big Bounce" universe ($\Omega_{\Lambda,0} = 1.8$, $\kappa = +1$).

TABLE 6.2 Properties of the Benchmark Model

	List of Ingredients	
photons:	$\Omega_{\gamma,0} = 5.0 \times 10^{-5}$	
neutrinos:	$\Omega_{\nu,0} = 3.4 \times 10^{-5}$	
total radiation:	$\Omega_{r,0} = 8.4 \times 10^{-5}$	
baryonic matter:	$\Omega_{\text{bary},0} = 0.04$	
nonbaryonic dark matter:	$\Omega_{\text{dm},0} = 0.26$	
total matter:	$\Omega_{m,0} = 0.30$	
cosmological constant:	$\Omega_{\Lambda,0} \approx 0.70$	
	Important Epochs	
radiation-matter equality:	$a_{rm} = 2.8 \times 10^{-4}$	$t_{rm} = 4.7 \times 10^4$ yr
matter-lambda equality:	$a_{m\Lambda} = 0.75$	$t_{m\Lambda} = 9.8$ Gyr
Now:	$a_0 = 1$	$t_0 = 13.5$ Gyr

matter is roughly six times greater: $\Omega_{\text{dm},0} \approx 0.26$. The bulk of the energy density in the Benchmark Model, however, is not provided by radiation or matter, but by a cosmological constant, with $\Omega_{\Lambda,0} = 1 - \Omega_{m,0} - \Omega_{r,0} \approx 0.70$.

The Benchmark Model was first radiation-dominated, then matter-dominated, and is now entering into its lambda-dominated phase. As we've seen, radiation gave way to matter at a scale factor $a_{rm} = \Omega_{r,0}/\Omega_{m,0} = 2.8 \times 10^{-4}$, correspond-

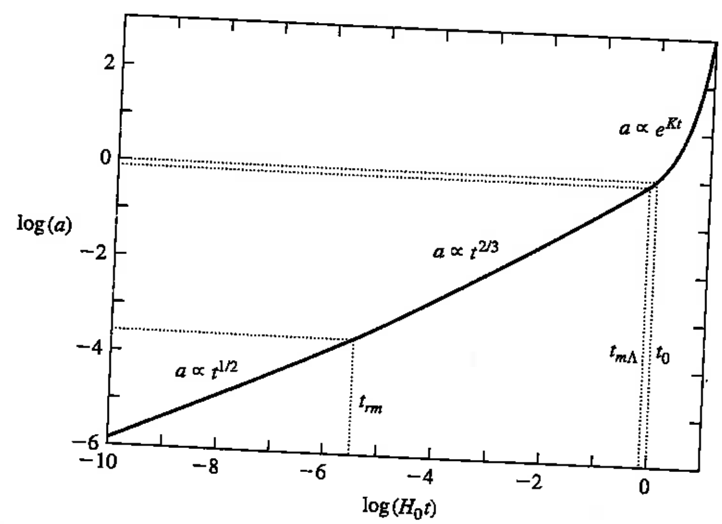

FIGURE 6.5 The scale factor a as a function of time t (measured in units of the Hubble time), computed for the Benchmark Model. The dotted lines indicate the time of radiation-matter equality, $a_{rm} = 2.8 \times 10^{-4}$, the time of matter-lambda equality, $a_{m\Lambda} = 0.75$, and the present moment, $a_0 = 1$.

ing to a time $t_{rm} = 4.7 \times 10^4$ yr. Matter, in turn, gave way to the cosmological constant at $a_{m\Lambda} = (\Omega_{m,0}/\Omega_{\Lambda,0})^{1/3} = 0.75$, corresponding to $t_{m\Lambda} = 9.8$ Gyr. The current age of the universe, in the Benchmark Model, is $t_0 = 13.5$ Gyr.

With $\Omega_{r,0}$, $\Omega_{m,0}$, and $\Omega_{\Lambda,0}$ known, the scale factor $a(t)$ can be computed numerically using the Friedmann equation, in the form of equation (6.6). Figure 6.5 shows the scale factor, thus computed, for the Benchmark Model. Note that the transition from the $a \propto t^{1/2}$ radiation-dominated phase to the $a \propto t^{2/3}$ matter-dominated phase is not an abrupt one; neither is the later transition from the matter-dominated phase to the exponentially growing lambda-dominated phase. One curious feature of the Benchmark Model illustrated vividly in Figure 6.5 is that we are living very close to the time of matter-lambda equality.

Once $a(t)$ is known, other properties of the Benchmark Model can be computed readily. For instance, the upper panel of Figure 6.6 shows the current proper

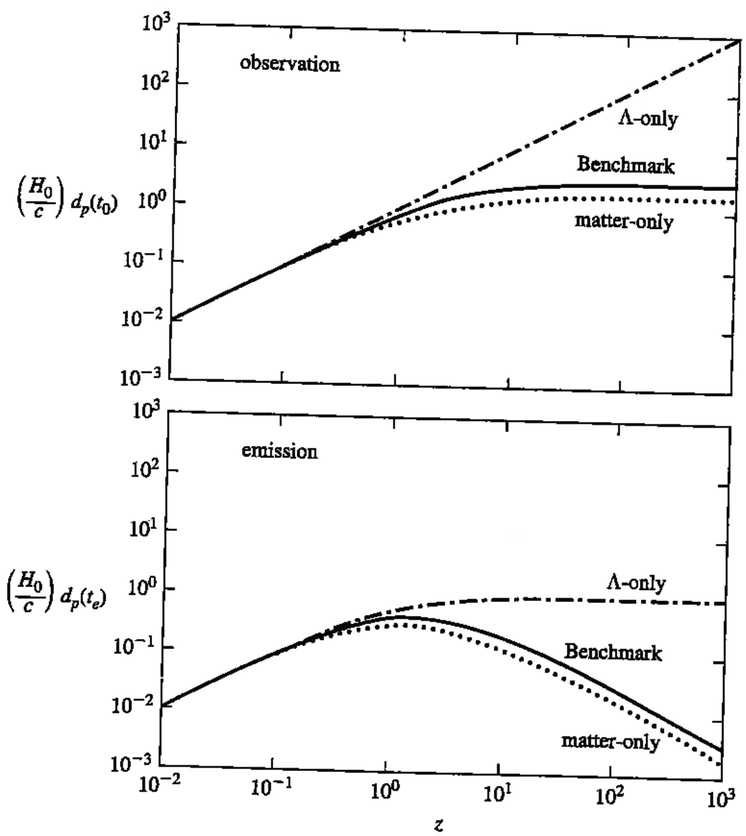

FIGURE 6.6 The proper distance to a light source with observed redshift z. The upper panel shows the distance at the time of observation; the lower panel shows the distance at the time of emission. The bold solid line indicates the Benchmark Model, the dot-dash line a flat, lambda-only universe, and the dotted line a flat, matter-only universe.

distance to a galaxy with redshift z. The heavy solid line is the result for the Benchmark Model; for purposes of comparison, the result for a flat lambda-only universe is shown as a dot-dash line and the result for a flat matter-only universe is shown as the dotted line. In the limit $z \to \infty$, the proper distance $d_p(t_0)$ approaches a limiting value $d_p \to 3.24c/H_0$, in the case of the Benchmark Model. Thus, the Benchmark Model has a finite horizon distance,

$$d_{\rm hor}(t_0) = 3.24c/H_0 = 3.12ct_0 = 14{,}000\,{\rm Mpc}. \tag{6.42}$$

If the Benchmark Model is a good description of our own universe, then we can't see objects more than 14 gigaparsecs away because light from them has not yet had time to reach us. The lower panel of Figure 6.6 shows $d_p(t_e)$, the distance to a galaxy with observed redshift z at the time the observed photons were emitted. For the Benchmark Model, $d_p(t_e)$ has a maximum for galaxies with redshift $z = 1.6$, where $d_p(t_e) = 0.41c/H_0$.

When astronomers observe a distant galaxy, they ask the related, but not identical, questions, "How far away is that galaxy?" and "How long has the light from that galaxy been traveling?" In the Benchmark Model, or any other model, we can answer the question "How far away is that galaxy?" by computing the proper distance $d_p(t_0)$. We can answer the question "How long has the light from that galaxy been traveling?" by computing the *lookback time*. If light emitted at time t_e is observed at time t_0, the lookback time is simply $t_0 - t_e$. In the limits of very small redshifts, $t_0 - t_e \approx z/H_0$. However, as shown in Figure 6.7, at larger redshifts the relation between lookback time and redshift becomes nonlin-

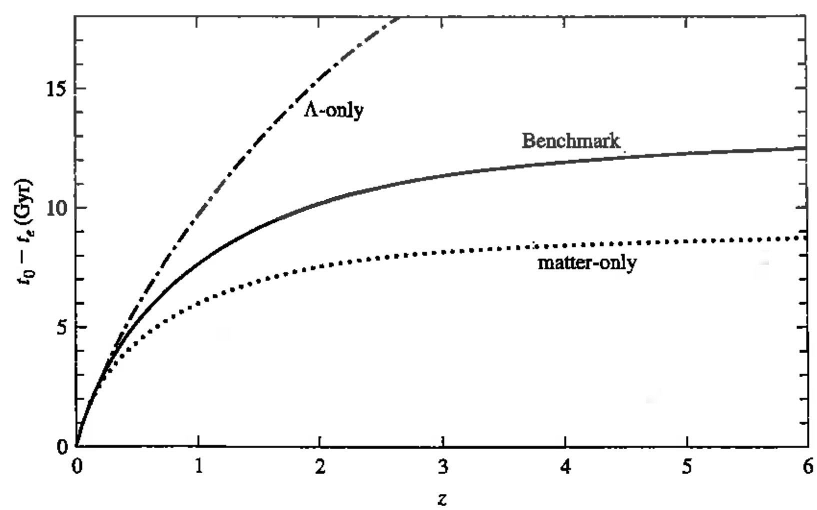

FIGURE 6.7 The lookback time, $t_0 - t_e$, for galaxies with observed redshift z. The Hubble time is assumed to be $H_0^{-1} = 14$ Gyr. The heavy solid line shows the result for the Benchmark Model, the dot-dash line for a flat, lambda-only universe, and the dotted line for a flat, matter-only universe.