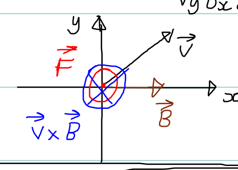
AN ION OF (NEGATIVE) CHARGE $-10^{-7}C$ MOVES WITH VELOCITY $\vec{v} = (10^{7}\hat{\imath} + 10^{7}\hat{\jmath})\,m/s$ THROUGH A MACHINE THAT IS CAPABLE OF PRODUCING A 1.0T MAGNETIC FIELD IN THE $\hat{\imath}, \hat{\jmath}$ AND $\hat{k}$ DIRECTION. WHAT IS THE FORCE ON THE PARTICLE IF THE MACHINE IS TURNED ON TO PRODUCE (a) $\vec{B} = 1.0T\,\hat{\imath}$? (b) $\vec{B} = 1.0T\,\hat{\jmath}$? (c) $\vec{B} = 1.0T\,\hat{k}$ (d) ALL THREE OF (a), (b) AND (c) ARE APPLIED AT ONCE? IN EACH CASE, SKETCH THE DIRECTION OF $\vec{v}, \vec{B}, \vec{v}\times\vec{B}$ AND THE FORCE, $\vec{F}$

(a) $$\vec{v}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ V_x & V_y & V_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ V_x & V_y & 0 \\ B_x & 0 & 0 \end{vmatrix} = \hat{\imath}\begin{vmatrix} V_y & 0 \\ 0 & 0 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} V_x & 0 \\ B_x & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} V_x & V_y \\ B_x & 0 \end{vmatrix} = 0\hat{\imath} - 0\hat{\jmath} + (0V_x - V_y B_x)\hat{k}$$

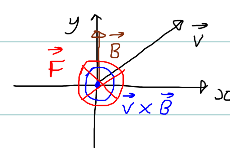
$$= -V_y B_x \hat{k} = -10^{7} m/s\, 1.0T\,\hat{k} = -10^{7} Tm/s\,\hat{k}$$

$$\vec{F} = q(\vec{v}\times\vec{B}) = (-10^{-7}C)(-10^{7} Tm/s)\hat{k} = \underline{\underline{1.0N\,\hat{k}}}$$



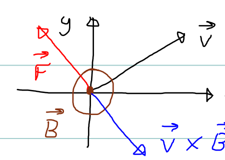
(b) $$\vec{v}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ V_x & V_y & 0 \\ 0 & B_y & 0 \end{vmatrix} = \hat{\imath}\begin{vmatrix} V_y & 0 \\ B_y & 0 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} V_x & 0 \\ 0 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} V_x & V_y \\ 0 & B_y \end{vmatrix} = V_x B_y \hat{k} = 10^{7} m/s\, 1.0T\,\hat{k} = 10^{7} Tm/s\,\hat{k}$$

$$\vec{F} = q(\vec{v}\times\vec{B}) = (-10^{-7}C)10^{7} Tm/s = \underline{\underline{-1.0N\,\hat{k}}}$$



(c) $$\vec{v}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ V_x & V_y & 0 \\ 0 & 0 & B_z \end{vmatrix} = \hat{\imath}\begin{vmatrix} V_y & 0 \\ 0 & B_z \end{vmatrix} - \hat{\jmath}\begin{vmatrix} V_x & 0 \\ 0 & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} V_x & V_y \\ 0 & 0 \end{vmatrix} = V_y B_z \hat{\imath} - V_x B_z \hat{\jmath} = 10^{7} Tm/s(\hat{\imath} - \hat{\jmath})$$

$$\vec{F} = q(\vec{v}\times\vec{B}) = (-10^{-7}C)(10^{7} Tm/s)(\hat{\imath} - \hat{\jmath}) = \underline{\underline{(-1.0\hat{\imath} + 1.0\hat{\jmath})\,N}}$$



NOTE! $\vec{v}\times\vec{B}$ AND $\vec{F}$ ARE ALWAYS AT 90° TO $\vec{v}$

(d) THERE ARE TWO EQUIVALENT APPROACHES... BECAUSE FIELDS AND FORCES BOTH OBEY SUPERPOSITION, YOU COULD FIND THE VECTOR SUM OF THE THREE RESULTS FOR THE FORCE FROM (a), (b) AND (c), OR YOU COULD WORK WITH THE VECTOR SUM OF THE THREE

B-FIELDS. LET'S TRY BOTH...

THE COMBINED B-FIELD IS $\vec{B} = 1.0T\hat{\imath} + 1.0T\hat{\jmath} + 1.0T\hat{k}$

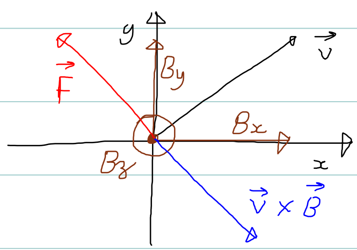
$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ V_x & V_y & 0 \\ B_x & B_y & B_z \end{vmatrix} = \hat{\imath}\begin{vmatrix} V_y & 0 \\ B_y & B_z \end{vmatrix} - \hat{\jmath}\begin{vmatrix} V_x & 0 \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} V_x & V_y \\ B_x & B_y \end{vmatrix} = V_y B_z \hat{\imath} - V_x B_z \hat{\jmath} + (V_x B_y - V_y B_x)\hat{k}$$

$$= 10^7 Tm/s\,\hat{\imath} - 10^7 Tm/s\,\hat{\jmath} + (10^7 Tm/s - 10^7 Tm/s)\hat{k} = 10^7 Tm/s(\hat{\imath} - \hat{\jmath})$$

$$\vec{F} = q(\vec{v}\times\vec{B}) = (-10^{-7}C)(10^7 Tm/s)(\hat{\imath}-\hat{\jmath}) = -(1.0\hat{\imath} - 1.0\hat{\jmath})N = \underline{\underline{(-1.0\hat{\imath} + 1.0\hat{\jmath})N}}$$

ALTERNATIVELY, ADD THE FORCES FROM (a), (b) AND (c):

$$\vec{F} = 1.0N\hat{k} - 1.0N\hat{k} + (-1.0\hat{\imath} + 1.0\hat{\jmath})N = \underline{\underline{(-1.0\hat{\imath} + 1.0\hat{\jmath})N}}$$



COUNTER-INTUITIVELY, B_x AND B_y <u>CANCEL</u> IN THIS SITUATION. CONSIDER THIS

$\vec{B} = 1.0T\hat{\imath} + 1.0T\hat{k}$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ V_x & V_y & 0 \\ B_x & B_y & 0 \end{vmatrix} = \hat{\imath}\begin{vmatrix} V_y & 0 \\ B_y & 0 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} V_x & 0 \\ B_x & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} V_x & V_y \\ B_x & B_y \end{vmatrix} = (V_x B_y - V_y B_x)\hat{k}$$

BECAUSE $V_x = V_y$ AND $B_x = B_y$

$\vec{V} \times \vec{B} = 0$!