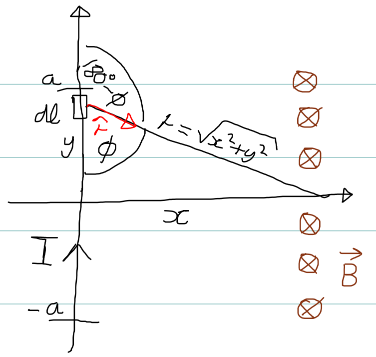
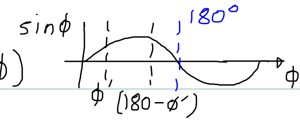


$\sin\phi = \sin(180^\circ - \phi)$

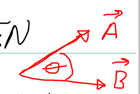


$$\Rightarrow \sin(180-\phi) = \sin\phi = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}}$$

$$\boxed{\text{BIOT-SAVART LAW} \quad d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\ell} \times \hat{r}}{r^2}}$$

$$\Rightarrow d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{|d\vec{\ell}||\hat{r}|\sin(180^\circ-\phi)}{r^2}$$

WHERE $(180^\circ - \phi)$ IS THE ANGLE BETWEEN $d\vec{\ell}$ AND $\hat{r}$ AND THE CROSS PRODUCT STATES $\vec{A} \times \vec{B} = |A||B|\sin\theta$



NOTE THAT $|\hat{r}| = 1$ AS $\hat{r}$ IS A UNIT VECTOR

$r^2 = x^2 + y^2$

$$\Rightarrow d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\ell}{r^2}\frac{x}{(x^2+y^2)^{1/2}} = \frac{\mu_0 I}{4\pi}\frac{x\,d\ell}{(x^2+y^2)^{3/2}}$$

ALL $d\vec{\ell}$ CONTRIBUTIONS ALONG y

WE NEED TO SUPERPOSE (SUM) ALL CONTRIBUTIONS TO $\vec{B}$ ALONG THE WIRE FROM $-a$ TO a

$$\vec{B} = \int_{y=-a}^{y=a} d\vec{B} = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x\,dy}{(x^2+y^2)^{3/2}} = \frac{\mu_0 I}{2\pi x}\frac{a}{\sqrt{x^2+a^2}}$$ (DIRECTION GIVEN BY RIGHT HAND RULE)

THE INTEGRATION STEP IS NOT OBVIOUS... SEE IntegrationForBfieldDueToWire.pdf FOR MORE DETAILS

CLOSE TO A LONG WIRE $a \gg x$, SO $x^2 + a^2 \simeq a^2$ AND $\frac{a}{\sqrt{x^2+a^2}} \simeq 1$

THEN $\vec{B} = \frac{\mu_0 I}{2\pi x}$ (DIRECTION OF B-FIELD GIVEN BY RIGHT-HAND RULE) WHERE x IS THE DIRECTION PERPENDICULAR TO THE WIRE