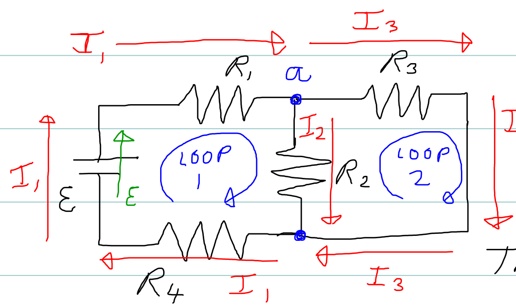


$R_1 = R_2 = 20\,\Omega$, $R_3 = 30\,\Omega$, $R_4 = 80\,\Omega$

IF $\mathcal{E} = 12V$ (a) USE KIRCHHOFF'S LAWS TO FIND THE CURRENT THROUGH THE BATTERY (b) SIMPLIFY PART OF THE CIRCUIT USING THE RULES FOR RESISTORS IN PARALLEL AND CHECK YOUR RESULT FOR PART (a), (c) IF $R_1 = R_2 = R_3 = R_4$ WHICH ONE RESISTOR SHOULD BE REMOVED TO MAXIMIZE THE LIFETIME OF THE BATTERY?

(a) THE CIRCUIT IS LABELLED SO THAT WE SEE $\mathcal{E}\uparrow$ THE DIRECTION OF INCREASING EMF IN THE BATTERY ↻ THE KIRCHHOFF LOOPS IN A CONSISTENT DIRECTION (CLOCKWISE) AND $\rightarrow$ THE CURRENT FLOW. THE ONLY CONSCIOUS CHOICE FOR THE CURRENT FLOW IS THAT THERE IS NO JUNCTION THAT HAS ALL CURRENT FLOWING IN $\rightarrow \bullet \leftarrow$ AND NO JUNCTION THAT HAS ALL CURRENT FLOWING OUT $\leftarrow \bullet \rightarrow$. WE WILL WRITE DOWN 3 EQUATIONS BASED ON THE LOOPS AND JUNCTION a:

LOOP 1: $\sum \Delta V = 0 \Rightarrow \mathcal{E} - I_1R_1 - I_2R_2 - I_1R_4 = 0 \Rightarrow I_2 = \dfrac{\mathcal{E} - I_1R_1 - I_1R_4}{R_2} = \dfrac{\mathcal{E} - I_1(R_1+R_4)}{R_2}$

LOOPED AGAINST I_2 — SUBBED IN FOR I_2

LOOP 2: $\sum \Delta V = 0 \Rightarrow -I_3R_3 + I_2R_2 \Rightarrow I_3 = \dfrac{I_2R_2}{R_3} = \dfrac{\mathcal{E}\cancel{R_2} - I_1(R_1+R_4)\cancel{R_2}}{R_3\cancel{R_2}} = \dfrac{\mathcal{E} - I_1(R_1+R_4)}{R_3}$

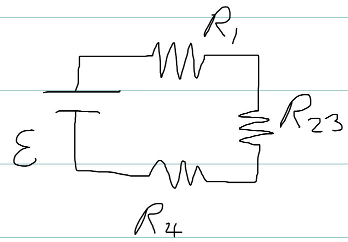
LOOP 1 — LOOP 2

JUNCTION a: $I_1 = I_3 + I_2 \Rightarrow I_1 = \dfrac{\mathcal{E} - I_1(R_1+R_4)}{R_2} + \dfrac{\mathcal{E} - I_1(R_1+R_4)}{R_3} = \dfrac{R_3\mathcal{E} - R_3I_1(R_1+R_4)}{R_3R_2} + \dfrac{R_2\mathcal{E} - R_2I_1(R_1+R_4)}{R_2R_3}$

$\Rightarrow I_1R_2R_3 + I_1R_3(R_1+R_4) + I_1R_2(R_1+R_4) = \mathcal{E}(R_2+R_3) \Rightarrow I_1 = \dfrac{\mathcal{E}(R_2+R_3)}{R_2R_3 + R_3(R_1+R_4) + R_2(R_1+R_4)} = \dfrac{\mathcal{E}(R_2+R_3)}{R_2R_3 + (R_3+R_2)(R_1+R_4)}$

SO, SUBBING IN THE NUMBERS, $I_1 = \dfrac{12(20+30)}{20.30 + (30+20)(20+80)} = \dfrac{12.50}{600+50.100} = \dfrac{6\cancel{0}\cancel{0}}{56\cancel{0}\cancel{0}} = \underline{\underline{0.107A}}$

(b) R_2 AND R_3 ARE IN PARALLEL, AS THE CURRENT CAN FLOW THROUGH EITHER SO THE CIRCUIT CAN BE REDUCED TO:



WHERE $\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} \Rightarrow \frac{1}{R_{23}} = \frac{R_2 + R_3}{R_2 R_3}$

$$\Rightarrow R_{23} = \frac{R_2 R_3}{(R_2+R_3)} = \frac{20 \cdot 30}{50} = \frac{600}{50} = 12\,\Omega$$

NOW THERE IS JUST A SINGLE LOOP, $\mathcal{E} - I_1 R_1 - I_1 R_{23} - I_1 R_4 = 0$

$$\Rightarrow I_1 = \frac{\mathcal{E}}{(R_1 + R_{23} + R_4)} = \frac{12}{(20+12+80)} = \frac{6}{56} = \underline{\underline{0.107\,A}}$$

(c) $\text{POWER} = \frac{\text{ENERGY}}{\text{TIME}}$ TO MAXIMIZE TIME WE MUST MINIMIZE POWER

IMAGINE THE CIRCUIT IS MADE UP OF A SINGLE RESISTOR AND A SINGLE BATTERY OF $\mathcal{E} = 12\,V$. AS WE CHANGE THE RESISTANCE OF THAT SINGLE RESISTOR, THE CURRENT RUNNING THROUGH THE CIRCUIT WILL CHANGE, BUT THE VOLTAGE WILL BE THE SAME AT $\mathcal{E} = 12\,V$. THE SINGLE RESISTANCE IS R_{EQ} AND THE POWER IS $P = \frac{\mathcal{E}^2}{R_{EQ}}$

SO, TO MINIMIZE POWER WE MAXIMIZE R_{EQ}

SERIES   PART (b)

$$R_{EQ} = R_1 + R_{23} + R_4 \quad \text{WHERE } R_{23} = \frac{R_2 R_3}{(R_2+R_3)} \Rightarrow R_{EQ} = R_1 + \frac{R_2 R_3}{(R_2+R_3)} + R_4$$

REMEMBER, $R_1 = R_2 = R_3 = R_4$

IF WE REMOVE R_1 OR R_4, $R_{EQ} = R + \frac{RR}{2R} = R + \frac{R}{2} = \frac{3R}{2}$

BECAUSE ALL 3 RESISTORS WILL BE IN SERIES

$R_{EQ} = R_1 + R_{23} + R_4$ WHEN R_2 OR R_3 IS REMOVED $R_{23} = R$

$$\Rightarrow R_{EQ} = R + R + R = 3R$$

AS MAXIMIZING R_{EQ} MAXIMIZES THE BATTERIES LIFETIME, AND $3R > \frac{3R}{2}$, WE SHOULD REMOVE $\underline{\underline{R_2}}$ $\underline{\underline{\text{OR}}}$ $\underline{\underline{R_3}}$