$$\boxed{B = \frac{\mu_0 I}{4\pi} \int_{y=-a}^{y=a} \frac{x\,dy}{(x^2+y^2)^{3/2}}}$$

SET $A = \int \frac{x\,dy}{(x^2+y^2)^{3/2}}$ FOR THE INTEGRATION

LET $y = x\tan\theta \Rightarrow dy = x\sec^2\theta\,d\theta$ (DERIVATIVE OF $\tan\theta = \sec^2\theta$)

$$\Rightarrow A = \int \frac{x^2\sec^2\theta\,d\theta}{(x^2+x^2\tan^2\theta)^{3/2}} = \int \frac{x^2\sec^2\theta\,d\theta}{(x^2(1+\tan^2\theta))^{3/2}} = \int \frac{x^2\sec^2\theta\,d\theta}{(x^2\sec^2\theta)^{3/2}}$$

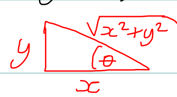
$1 + \tan^2\theta = \sec^2\theta$

$$\Rightarrow A = \int \frac{x^2\sec^2\theta\,d\theta}{x^3\sec^3\theta} = \frac{1}{x}\int\frac{d\theta}{\sec\theta} = \frac{1}{x}\int\cos\theta\,d\theta = \frac{1}{x}\left[\sin\theta\right]$$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \int\cos\theta\,d\theta = \sin\theta$

OUR INTEGRATION IS OVER $y=-a$ TO $y=a$, SO WE NEED TO CONVERT $\sin\theta$ BACK TO y

WE LET $y = x\tan\theta \Rightarrow \tan\theta = \frac{y}{x}$



$\Rightarrow \sin\theta = \frac{y}{\sqrt{x^2+y^2}}$

SO, $A = \frac{1}{x}\left[\frac{y}{\sqrt{x^2+y^2}}\right]$

SUBSTITUTE BACK IN FOR A TO FIND THE B-FIELD...

$$B = \frac{\mu_0 I}{4\pi} A\Big|_{y=-a}^{y=a} = \frac{\mu_0 I}{4\pi x}\left[\frac{y}{\sqrt{x^2+y^2}}\right]_{y=-a}^{y=a} = \frac{\mu_0 I}{4\pi x}\left[\frac{a}{\sqrt{x^2+a^2}} - \frac{-a}{\sqrt{x^2+a^2}}\right] = \frac{\mu_0 I}{4\pi x}\frac{2a}{\sqrt{x^2+a^2}}$$

$$\Rightarrow \underline{B = \frac{\mu_0 I}{2\pi x}\frac{a}{\sqrt{x^2+a^2}}}$$