THIS QUESTION CONCERNS ELECTRIC POTENTIAL AT ALL DISTANCES FROM A UNIFORMLY CHARGED INSULATING SPHERE THAT CARRIES A TOTAL POSITIVE CHARGE Q. (1) WRITE A BRIEF EXPLANATION OF HOW THE ELECTRIC FIELD BEHAVES WITH r IN AND OUT OF THE SPHERE. (2) DERIVE THE POTENTIAL $V(r)$ AT ALL DISTANCES FROM THE CENTER OF THE SPHERE TO INFINITY. ASSUME $V \to 0$ AS $r \to \infty$ (3) SKETCH THE POTENTIAL FOR THE SAME SITUATION WITH Q NEGATIVE.

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(1) THE SPHERE IS INSULATING SO THE CHARGE WON'T RUN TO THE SPHERE'S SURFACE BUT WILL BE UNIFORMLY SPREAD IN THE VOLUME (Q GOES AS r^3). GAUSS'S LAW SAYS THE E-FIELD GOES AS SURFACE AREA OR r^2 ($4\pi r^2 E$ GOES AS $\frac{Q}{\varepsilon_0}$ OR $\frac{r^3}{\varepsilon_0}$). SO THE ELECTRIC FIELD INSIDE THE SPHERE WILL GO AS r (E GOES AS $\frac{r^3}{4\pi r^2}$ SO E GOES AS r). OUTSIDE THE SPHERE THE TOTAL CHARGE ENCLOSED IS ALWAYS Q, SO $4\pi r^2 E$ GOES AS Q AND E GOES AS $\frac{1}{r^2}$

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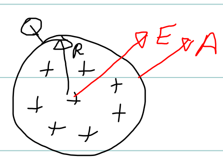
(2)



$\theta = 0$, $\cos\theta = 1$

at $r>R$: $\int \vec{E}\cdot d\vec{A} = \int E\,dA = E\int dA = \frac{Q}{\varepsilon_0}$ (ALL Q IS INSIDE)

$$\Rightarrow E\,4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{4\pi r^2 Q}{\varepsilon_0} = \frac{kQ}{r^2}$$

volume charge density ρ = charge per unit volume

at $r<R$: $Q = \rho V \Rightarrow dQ = \rho\, dV \Rightarrow Q(r) = \int_0^r \rho\, dV = \frac{4}{3}\pi r^3 \rho = \frac{\cancel{\frac{4}{3}\pi} r^3 Q}{\cancel{\frac{4}{3}\pi} R^3}$

So, $Q(r) = \frac{r^3 Q}{R^3}$

GAUSS'S LAW at $r<R$:

$$\int \vec{E}\cdot d\vec{A} = \int E\,dA = 4\pi r^2 E = \frac{Q}{\varepsilon_0} = \frac{r^3 Q}{\varepsilon_0 R^3} \Rightarrow E = \frac{r^3 Q}{\varepsilon_0 4\pi r^2 R^3} = \frac{Qr}{4\pi\varepsilon_0 R^3} = \frac{kQr}{R^3}$$

NOW FIND THE POTENTIAL

$r>R$: $V = \int_r^\infty \vec{E}\cdot d\vec{l} = -\int_\infty^r E\,dr = -\int_\infty^r \frac{kQ}{r^2} = \left[\frac{kQ}{r}\right]_\infty^r = \frac{kQ}{r} - \underset{\text{zero potential at }\infty}{0} = \boxed{\frac{kQ}{r}}$

$r<R$: $V = -\int_\infty^R \frac{kQ}{r^2}dr - \int_R^r \frac{kQr\,dr}{R^3} = \frac{kQ}{R} - \frac{kQ}{2R^3}\left[r^2 - R^2\right] = \boxed{\frac{kQ}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right]}$

So, at $\boxed{r > R,\ V(r) = \frac{kQ}{r}}$ and at $\boxed{r < R,\ V(R) = \frac{kQ}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right]}$

NOTE ALSO THAT AT $r < R$ $E(r)$ GOES AS r, AS WE WROTE IN PART (1)

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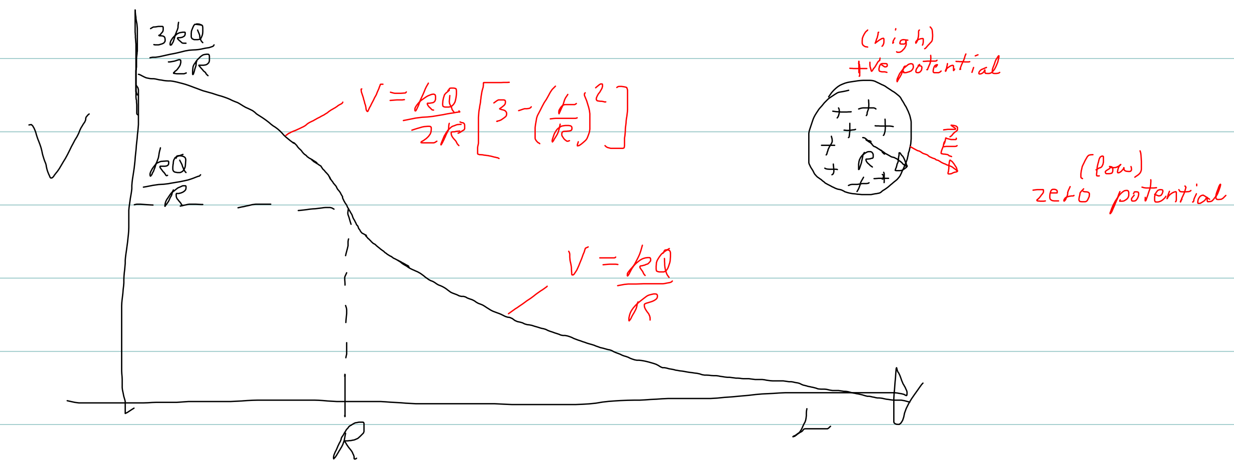
(3) SKETCH WHAT WE DERIVED:



BUT, WHAT WAS REQUESTED WAS FOR A NEGATIVELY CHARGED SPHERE

THE E-FIELD POINTS FROM HIGH TO LOW POTENTIAL. IF $V=0$ AT $r=\infty$, THEN AS THE E-FIELD POINTS IN TOWARDS THE NEGATIVELY CHARGED SPHERE, THE POTENTIAL MUST DECREASE FROM ZERO POTENTIAL. HENCE: