THIS QUESTION CONCERNS ELECTRIC POTENTIAL AT ALL DISTANCES FROM A UNIFORMLY CHARGED INSULATING SPHERE THAT CARRIES A TOTAL POSITIVE CHARGE Q. (1) WRITE A BRIEF EXPLANATION OF HOW THE ELECTRIC FIELD BEHAVES WITH r IN AND OUT OF THE SPHERE. (2) DERIVE THE POTENTIAL $V(r)$ AT ALL DISTANCES FROM THE CENTER OF THE SPHERE TO INFINITY. ASSUME $V \to 0$ AS $r \to \infty$ (3) SKETCH THE POTENTIAL FOR THE SAME SITUATION WITH Q NEGATIVE.

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(1) ESSENTIALLY ALL QUESTIONS IN ELECTROSTATICS START WITH DERIVING THE E-FIELD. ALL PROBLEMS WITH CONTINUOUS CHARGE START BY WRITING DOWN THE CHARGE IN TERMS OF THE (LENGTH, SURFACE AREA OR VOLUME) CHARGE DENSITY. (a) WRITE DOWN THE Q IN TERMS OF THE APPROPRIATE CHARGE DENSITY (b) WRITE DOWN GAUSS'S LAW TO RELATE THE E-FIELD TO THE CHARGE (AND HENCE THE CHARGE DENSITY (c) HOW DOES THE E-FIELD AT $r>R$ (OUTSIDE THE SPHERE) DEPEND ON CHARGE (d) REMEMBER THAT THIS IS AN INSULATOR, SO CHARGE WON'T FLOW TO THE SURFACE. SO FIND AN EXPRESSION FOR HOW Q GOES AS r (i.e. $Q(r)$) IN TERMS OF CHARGE DENSITY. IF $4\pi r^2 E = \frac{Q}{\varepsilon_0}$ THEN E GOES AS $\frac{Q(r)}{r^2}$

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(2) (a) WRITE DOWN AN EXPRESSION FOR E AT $r>R$ (OUTSIDE THE SPHERE) AND AT $r<R$ (INSIDE THE SPHERE)

(b) $\Delta V_{a\,to\,b} = \int_a^b \vec{E}\cdot d\vec{l}$ SKETCH THE SITUATION TO SHOW THAT $\Delta V = \int_a^b E\,dr$

(c) AT $r>R$ $\Delta V = \int_r^\infty \underbrace{E}_{E_{OUTSIDE}}\,dr$ OR $-\int_\infty^r \underbrace{E}_{E_{OUTSIDE}}\,dr$

(d) AT $r<R \Rightarrow \Delta V = -\int_R^\infty \underbrace{E}_{E_{OUTSIDE}}\,dr - \int_R^r \underbrace{E}_{E_{INSIDE}}\,dr$ WHERE WE HAVE TO SPLIT THINGS UP INTO TWO PARTS BECAUSE THE E-FIELD CHANGES INSIDE THE SPHERE

((NOTE THAT $\Delta V_{0\,to\,\infty} = \Delta V_{0\,to\,r} + \Delta V_{r\,to\,\infty}$ BECAUSE $(V_0 - V_\infty) = (V_0 - V_r) + (V_r - V_\infty)$))

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(3) DRAW $V(r)$ FROM (2). WHAT WILL CHANGE FOR A NEGATIVELY CHARGED SPHERE? WILL $\vec{E}$ BE IN THE SAME DIRECTION? DOES $\vec{E}$ POINT FROM LOW-TO-HIGH OR FROM HIGH-TO-LOW POTENTIAL?