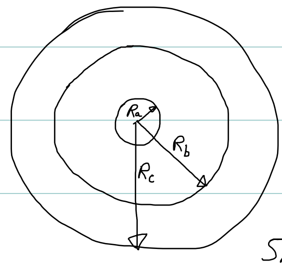

A SOLID CONDUCTING SPHERE OF RADIUS R_a IS PLACED INSIDE A CONDUCTING SPHERICAL SHELL OF INNER RADIUS R_b AND OUTER RADIUS R_c. THE INNER SPHERE HAS CHARGE $+5Q$ nC AND THE OUTER SHELL HAS UNIFORM VOLUME CHARGE DENSITY $\rho = \frac{+10Q}{\frac{4}{3}\pi(R_c^3 - R_b^3)}$ nCm^{-3}

FIND THE ELECTRIC FIELD AND POTENTIAL AT $r < R_a$, $R_a < r < R_b$, $R_b < r < R_c$ AND $r > R_c$

---

(1) THE E-FIELD

THE TWO STARTING POINTS FOR ELECTROSTATICS PROBLEMS ARE ALMOST ALWAYS (i) FIND THE E-FIELD and (ii) IF I HAVE A CONTINUOUS CHARGE DISTRIBUTION, RELATE THE CHARGE TO THE CHARGE DENSITY (USUALLY DENOTED λ, σ or ρ). FOR THIS EXAMPLE FIND THE TOTAL CHARGE ON THE SPHERICAL SHELL AS $q = \rho V$ (Volume, not potential!). TO DO THIS WE NEED THE VOLUME OF THE SHELL... $V = \frac{4}{3}\pi R_c^3 - \frac{4}{3}\pi R_b^3 = \frac{4}{3}\pi(R_c^3 - R_b^3) \Rightarrow q = \rho V = +10Q$

NOW, FIND THE E-FIELD:

GAUSS'S LAW $\int \vec{E}\cdot d\vec{A} = \frac{Q_{IN}}{\varepsilon_0}$, $Q=0 \Rightarrow E=0$

$r < R_a$: THE CHARGE ON A CONDUCTOR MOVES TO THE SURFACE SO $q = 0$ AT $r < R_a \Rightarrow \underline{E = 0}$

θ between $E, dA = 0 \Rightarrow \cos\theta = 1$

$R_a < r < R_b$: $Q_{IN} = +5Q \Rightarrow \int \vec{E}\cdot d\vec{A} = E\int dA = E4\pi r^2 = \frac{5Q}{\varepsilon_0} \Rightarrow \underline{E = \frac{5Qk}{r^2}}$

$R_b < r < R_c$: INSIDE A CONDUCTOR CHARGE REDISTRIBUTES ITSELF SO $E = 0$ (SOMETIMES CALLED electrostatic equilibrium) SO $-5Q$ WILL RUN TO THE INNER SURFACE OF THE SHELL, $\underline{E = 0}$

$r > R_c$: $Q_{IN} = 15Q$ (DISTRIBUTED AS $+15Q$ ON THE OUTER SHELL SURFACE $-5Q$ ON THE INNER SHELL SURFACE $+5Q$ ON THE INNER BALL $\Rightarrow E = \frac{15Qk}{r^2}$

---

(2) THE POTENTIAL

THE POTENTIAL IS DEFINED AS $\Delta V = V_a - V_b = \int \vec{E}\cdot d\vec{l}$ OR $V_b - V_a = -\int_b^a \vec{E}\cdot d\vec{l}$ MOVING IN TOWARDS THE SOURCE OF THE E-FIELD. TO FIND THE POTENTIAL AT EACH DISTANCE, INTEGRATE IN FROM ZERO POTENTIAL ($V = 0$ at $r = \infty$)

AND SUM THE CONTRIBUTIONS FROM THE E-FIELD AT EACH DISTANCE:

$$\underline{r > R_C}: \quad V = -\int_{\infty}^{r} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{r} E\,dr = -\int_{\infty}^{r} \frac{15Qk}{r^2}\,dr = \left[\frac{15Qk}{r}\right]_{\infty}^{r} = \underline{\frac{15Qk}{r}} - \cancel{\frac{15Qk}{\infty}}^{\to 0}$$

$E = 0$ at $R_b < r < R_c$

$$\underline{R_b < r < R_C}: \quad V = -\int_{\infty}^{R_c} E\,dr - \int_{R_c}^{r} E\,dr = -\int_{\infty}^{R_c} \frac{15kQ}{r^2}\,dr - \cancel{\int_{R_c}^{r} 0\,dr}^{\to 0} = \frac{15Qk}{R_c}$$

$$\underline{R_a < r < R_b}: \quad V = \int_{\infty}^{R_c} E\,dr - \int_{R_c}^{R_b} E\,dr - \int_{R_b}^{r} E\,dr = \frac{15Qk}{R_c} - \cancel{\int_{R_c}^{R_b} 0\,dr}^{\to 0} - \int_{R_b}^{r} \frac{5Qk}{r^2}\,dr$$

$E = \frac{5Qk}{r^2}$ at $R_a < r < R_b$

$$= \frac{15Qk}{R_c} + \left[\frac{5Qk}{r}\right]_{R_b}^{r} = \underline{\frac{15Qk}{R_c} + \frac{5Qk}{r} - \frac{5Qk}{R_b}}$$

$E = 0$ at $r < R_a$

$$\underline{r < R_a}: \quad V = \int_{\infty}^{R_c} \frac{15kQ}{r^2}\,dr - \int_{R_c}^{R_b} 0\,dr - \int_{R_b}^{R_a} \frac{5kQ}{r^2} - \int_{R_a}^{r} 0\,dr = \underline{\frac{15kQ}{R_c} + \frac{5kQ}{R_a} - \frac{5kQ}{R_b}}$$

NOTE THAT THERE IS A SECOND (COMPLETELY EQUIVALENT) WAY TO PICTURE THIS...
YOU COULD TAKE THE CONTRIBUTION FROM EACH COMPONENT (THE INNER SPHERE, THE INNER SHELL, AND THE OUTER SHELL) AND ADD THEM. FOR INSTANCE:

AT $r < R_A$: $V_{INNER\ SPHERE} = \frac{5kQ}{R_a}$, $V_{INNER\ SHELL} = \frac{-5kQ}{R_b}$, $V_{OUTER\ SHELL} = \frac{15kQ}{R_c}$

WHERE I'VE PREVIOUSLY (IN LECTURE) SHOWN THAT THE POTENTIAL INSIDE A SPHERE ($r < R$) IS $\frac{kq}{R}$ AND THE POTENTIAL OUTSIDE A SPHERE ($r > R$) IS $\frac{kq}{r}$

So, $\underline{r < R_A}$: $V = V_{INNER\ SPHERE} + V_{INNER\ SHELL} + V_{OUTER\ SHELL} = \underline{\frac{5kQ}{R_a} - \frac{5kQ}{R_b} + \frac{15kQ}{R_c}}$

and AT $\underline{r > R_c}$: $V_{INNER\ SPHERE} = \frac{5kQ}{r}$, $V_{INNER\ SHELL} = -\frac{5kQ}{r}$, $V_{OUTER\ SHELL} = \frac{15kQ}{r}$

$$\Rightarrow V = V_{INNER\ SPHERE} + V_{INNER\ SHELL} + V_{OUTER\ SHELL} = \frac{5kQ}{r} - \frac{5kQ}{r} + \frac{15kQ}{r} = \underline{\frac{15kQ}{r}}$$

YOU MAY WANT TO VERIFY THAT YOU GET THE SAME ANSWERS AT $R_b < r < R_c$ AND AT $R_a < r < R_b$, BUT, YOU WILL AS IT'S JUST RE-VISUALIZING THE SAME PROBLEM