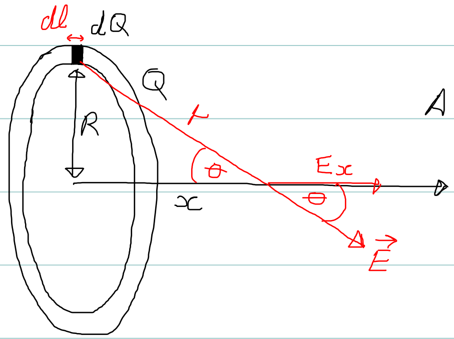

A VERY THIN RING OF RADIUS R CARRIES A TOTAL CHARGE OF Q. THE AXIS OUT FROM THE CENTER OF THE RING PERPENDICULAR TO THE RADIUS IS THE x-AXIS (AS DRAWN TO THE LEFT)

(1) WHAT IS THE ELECTRIC FIELD A DISTANCE x ALONG THE x-AXIS? (2) WHAT IS THE ELECTRIC POTENTIAL A DISTANCE x ALONG THE x-AXIS?

WE ARE WORKING WITH CONTINUOUS CHARGE, SO FIRST WRITE DOWN THE CHARGE IN TERMS OF CHARGE DENSITY. HERE, WE HAVE A VERY THIN RING, SO WE'LL USE λ, THE 1-D CHARGE DENSITY: $Q = \lambda \ell \Rightarrow dQ = \lambda\, d\ell$. NOTE THAT, USING THIS DEFINITION, ℓ, THE LINE OF CHARGE IS THE CIRCUMFERENCE OF THE RING: $\ell = 2\pi R$

(1) THE ELECTRIC FIELD

AS WITH ESSENTIALLY ALL PROBLEMS IN ELECTROSTATICS, START WITH THE E-FIELD. AS WE ARE WORKING OUT A COMPONENT OF THE E-FIELD, FIRST DETERMINE THE MAGNITUDE $|E|$ AND THEN USE GEOMETRY TO WORK OUT $|E|\hat{i}$ (WHERE $\hat{i}$ IS IN THE x-DIRECTION):

$$|E| = \frac{kq}{r^2} \qquad E_x = |E|\cos\theta = |E|\frac{x}{r} = \frac{kqx}{r^3}$$

WHAT WE HAVE DETERMINED IS THE E-FIELD ON THE x-AXIS DUE TO SOME CHARGE ON THE RING. BUT TO FIND THE CHARGE DUE TO THE ENTIRE RING WE MUST INTEGRATE OVER ALL INFINITESIMAL CONTINUOUS CHARGES dQ ($= \lambda\, d\ell$, SEE ABOVE) THAT

CONTRIBUTE dE TO THE TOTAL E-FIELD:

$$dE_x = \frac{kx}{r^3}dQ = \frac{kx\lambda}{r^3}dl \Rightarrow E_x = \frac{kx\lambda}{r^3}\int dl$$

THE INTEGRATION LIMITS ARE ALONG THE ENTIRE LINE OF CHARGE, OR ALL OF THE WAY AROUND THE RING FROM 0 TO THE FULL CIRCUMFERENCE ($2\pi R$):

$$E_x = \frac{kx\lambda}{r^3}\int_0^{2\pi R} dl = \frac{kx\lambda 2\pi R}{r^3} = \frac{kxQ}{r^3} = \frac{kxQ}{(R^2+x^2)^{3/2}}\,\hat{i}$$

NEED THINGS IN TERMS OF x TO GET $E(x)$

WHERE WE USED THE FACT THAT $r^2 = x^2+R^2$, THAT THE 1-D CHARGE DENSITY DEPENDS ON THE TOTAL LINE (RING) OF CHARGE AS $\lambda = \frac{Q}{\ell} = \frac{Q}{2\pi R}$, AND THAT THE DIRECTION IS $\hat{i}$

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② THE ELECTRIC POTENTIAL

ONE VALID WAY TO DETERMINE THE POTENTIAL WOULD BE TO REPEAT THE PREVIOUS ANALYSIS USING $V = \frac{kQ}{r} \Rightarrow dV = \frac{k\,dQ}{r} \Rightarrow V = \frac{k}{r}\int dQ$

BUT, AS WE'VE DETERMINED $E(x)$ IT IS ALSO EASY TO USE $V = \int \vec{E}\cdot d\vec{l} = \int E_x\,dx$

AS IS CONVENTIONAL, LET'S SET $V=0$ AT $r=\infty \Rightarrow V = \int_x^\infty E_x\,dx$

$$\Rightarrow V = \int_x^\infty E_x\,dx = kQ\int_x^\infty \frac{x}{(R^2+x^2)^{3/2}}dx = 0 - \frac{-kQ}{(R^2+x^2)^{\frac{1}{2}}} = \frac{kQ}{(R^2+x^2)^{\frac{1}{2}}}$$

THE FINAL INTEGRATION MAY NOT BE OBVIOUS.... SO LET $u = R^2+x^2 \Rightarrow du = 2x\,dx$

$$\Rightarrow \int\frac{x\,dx}{(R^2+x^2)^{3/2}} = \int\frac{du}{2u^{3/2}} = \int\frac{1}{2}u^{-3/2}du = -\cancel{\tfrac{1}{2}}\cancel{2}u^{-\frac{1}{2}} = -\frac{1}{u^{1/2}} = -\frac{1}{(R^2+x^2)^{\frac{1}{2}}}$$