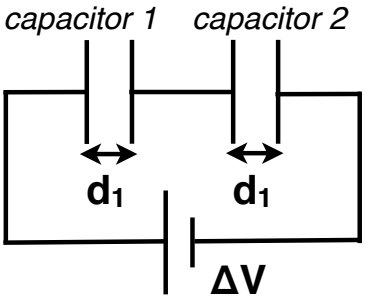
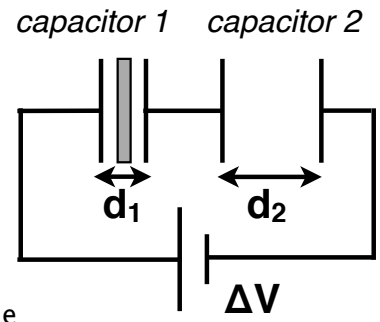
7. Capacitor Networks - II.



Two air-filled capacitors of plate area **A** and plate separation $\mathbf{d_1}$ are connected to a battery, as depicted to the right. For the purposes of this question, take the permittivity of air to be equal to $\boldsymbol{\varepsilon_o}$, the permittivity of free space.

a) Derive the equivalent capacitance of the circuit depicted in the upper diagram using **A**, $\mathbf{d_1}$ and $\boldsymbol{\varepsilon_o}$



b) Pokium has a dielectric constant of $\mathbf{K_p}$. A slab of pokium with a thickness of $\mathbf{d_1/3}$ and an area **A** (the same area as the capacitor plates) is placed in capacitor 1, as depicted to the right. Capacitor 1 now has a series of $\mathbf{d_1/3}$ of air, then $\mathbf{d_1/3}$ of pokium, then $\mathbf{d_1/3}$ of air between its plates. Find the equivalent capacitance of this new circuit using **A**, $\mathbf{d_1}$, $\mathbf{K_p}$, $\boldsymbol{\varepsilon_o}$ and $\mathbf{d_2}$, where $\mathbf{d_2}$ is the plate separation for capacitor 2.

c) $\mathbf{K_p}$ = 2.0 is the dielectric constant of pokium. If the two depicted circuits were set up to have the same equivalent capacitance, and $\mathbf{d_1}$ = 3.0cm, what would be the value of $\mathbf{d_2}$, the plate separation for capacitor 2, in cm?

d) $\boldsymbol{\varepsilon_o}$ = 8.9 x 10^{-12} F/m, **A** = 10.0 cm^2, and the battery supplies **ΔV** = 12 V. In the final configuration of the circuit depicted in the lower diagram, for $\mathbf{d_2}$ as calculated in part **(c)** of this problem, what is the stored energy in capacitor 2?

Hint: note that the equivalent capacitance was set up to be the same for the two depicted circuits.

(a)(i) WHAT IS THE EXPRESSION FOR CAPACITANCE BASED ON THE PHYSICAL CHARACTERISTICS OF A CAPACITOR? (ii) WHAT IS THE EXPRESSION FOR EQUIVALENT CAPACITANCE FOR MULTIPLE CAPACITORS IN SERIES (C_{EQ})?

(b) (i) ARE THE COMPONENTS OF CAPACITOR 1 IN SERIES OR IN PARALLEL? (ii) SO, WHAT IS AN EXPRESSION FOR C_1 BASED ON SUMMING IT'S COMPONENTS IN SERIES OR PARALLEL? (iii) WHAT IS AN EXPRESSION FOR C_2 (AS FOR PART (a))? (iv) SO, WHAT IS AN EXPRESSION FOR C_{EQ} FOR THE SECOND CIRCUIT?

(c) TO FIND WHEN THE CIRCUITS HAVE THE SAME C_{EQ}, EQUATE YOUR EQUATIONS FROM (a) AND (b). NOTE THAT IF $C_{EQ} = C'_{EQ}$ THEN $\frac{1}{C_{EQ}} = \frac{1}{C'_{EQ}}$ SO LEAVING YOUR EXPRESSIONS AS $\frac{1}{C_{EQ}} =$ MAY BE QUICKER

(d)(i) IN SERIES, IS THE CHARGE THE SAME ACROSS ALL ELEMENTS, OR IS IT THE VOLTAGE? (ii) FOR THE STORED ENERGY IN CAPACITOR 2 YOU WILL NEED TO FIND $U_2 = \frac{1}{2} C_2 V_2^2$ BUT V_2 MAY NOT EQUAL V ACROSS THE ENTIRE CIRCUIT (iii) FIND C_2 IN TERMS OF ITS PHYSICAL CHARACTERISTICS (LIKE IN PART (a)) (iv) USE C_{EQ} (ALSO CALCULATED IN PART (a)... SEE THE QUESTION HINT) TO FIND V_2 IF $C_2 = \frac{Q_2}{V_2}$ AND $C_{EQ} = \frac{Q}{V}$ (v) HENCE FIND U_2