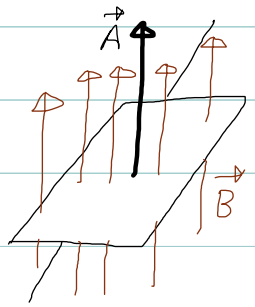
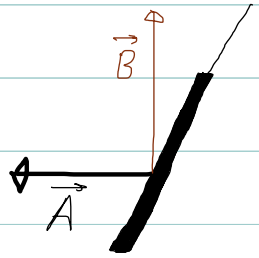
A MOTOR CONTAINS A LOOP THAT IS TURNING THROUGH A UNIFORM MAGNETIC FIELD. A LOOP TURNING THROUGH A UNIFORM MAGNETIC FIELD IS ALSO HOW A GENERATOR WORKS, AS THE CHANGING FLUX THROUGH THE LOOP ELECTROMAGNETICALLY INDUCES A CURRENT IN THE LOOP. FOR A MOTOR, THE INDUCED EMF IS UNDESIRABLE AS IT FIGHTS THE EMF THAT IS DRIVING THE MOTOR. FOR THIS REASON IT IS CALLED A "BACK" EMF. THE PURPOSE OF THIS QUESTION IS TO MODEL SUCH A BACK EMF.

A COIL CONSISTING OF N LOOPS OF AREA A TURNS AT A RATE OF ω DEGREES PER SECOND THROUGH A UNIFORM MAGNETIC FIELD OF STRENGTH B. THE LOOP IS ORIENTED SO THAT AT $\theta = 0°$ THE NORMAL VECTOR TO THE LOOP POINTS IN THE SAME DIRECTION AS THE B-FIELD AND AT $\theta = 90°$ THE NORMAL VECTOR TO THE LOOP POINTS AT 90° TO THE B-FIELD. (a) FIND AN EXPRESSION FOR HOW THE AREA THROUGH WHICH THE B-FIELD PASSES CHANGES WITH TIME, GIVEN THAT THE B-FIELD PASSES THROUGH THE ENTIRE AREA AT $t = 0s$ ($\theta = 0°$) AND THROUGH NONE OF THE AREA AT $\theta = 90°$. YOUR EXPRESSION SHOULD BE IN TERMS OF A, ω AND t WHERE $\theta = \omega t$ (i.e. WHERE THE ANGLE $\theta = 0°$ AT $t = 0$ AND $\theta = 360°$ AT TIME $t = \frac{360°}{\omega}$) (b) HENCE DERIVE AN EXPRESSION FOR THE INDUCED EMF (THE BACK EMF IN A MOTOR) IN TERMS OF A, B, N, ω AND t (c) A MOTOR CONSISTS OF 100 LOOPS OF AREA $10\ cm^2$ TURNING THROUGH A 0.1 T B-FIELD AT A RATE OF 1000° PER SECOND. WHAT IS THE MAXIMUM BACK EMF INDUCED IN THE MOTOR? (d) A MODIFICATION TO THE MOTOR IS SUGGESTED WHERE THE COIL DOES NOT ROTATE, BUT IS INSTEAD HELD AT $\theta = 0°$ (SO THE B-FIELD PASSES THROUGH THE ENTIRE AREA) AND MOVED LEFT, RIGHT, UP OR DOWN AT A VELOCITY $\vec{v} = 1000\ m/s$. WHAT EMF WOULD BE INDUCED IN THE COIL FOR THIS SET-UP?

(a)



$\theta = 0°$



$\theta = 90°$

AT $\theta = 0°$ THE AREA THROUGH WHICH THE B-FIELD PASSES IS $|A|$

AT $\theta = 90°$ THE AREA THROUGH WHICH THE B-FIELD PASSES IS ZERO, SO, WE NEED $A(\theta = 0°) = |A|$ AND $A(\theta = 90°) = 0$ (AND, e.g., $A(\theta = 180°) = -|A|$). A SUITABLE EXPRESSION WOULD BE $A(\theta) = |A| \cos\theta$. AS $\theta = \omega t$, $A = |A| \cos\omega t$

(b) FARADAY'S LAW STATES THAT THE INDUCED EMF IS $\mathcal{E} = -\frac{d\Phi_M}{dt}$ WHERE $\Phi_M = \oint \vec{B} \cdot d\vec{A}$. THE AREA THROUGH WHICH THE B-FIELD PASSES WAS FOUND IN (a) TO BE $\oint \vec{B} \cdot d\vec{A} = |B||A| \cos\omega t$ (NOTE THAT THIS SEEMS CORRECT FROM THE DEFINITION OF THE DOT PRODUCT, TOO... $\oint \vec{B} \cdot d\vec{A} = \oint |B||dA| \cos\theta = |B| \cos\theta \int |dA| = |B||A| \cos\theta = |B||A| \cos\omega t$), SO:

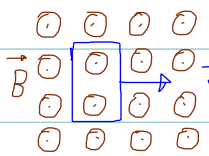
$\mathcal{E} = -\frac{d\Phi_M}{dt} = -\frac{d}{dt} |B||A| \cos\omega t = -|B||A| \omega \sin\omega t$ FOR ONE LOOP. FOR A COIL WITH N LOOPS THE AREA IS N TIMES LARGER AND $\mathcal{E} = -N|B||A|\omega \sin\omega t$ OR $|\mathcal{E}| = N|B||A|\omega|\sin\omega t|$

(c) THE MAXIMUM VALUE OF THE sin FUNCTION IS ONE, SO $|\mathcal{E}|_{MAX} = N|B||A|\omega$

PUTTING IN THE NUMBERS WE MUST TAKE CARE THAT $\omega = 1000°/s$ IS NOT IN THE SI UNIT OF RADIANS... $1000° = 1000° \times \frac{\pi}{180°}$ RADIANS $\Rightarrow \omega = 17.45\ s^{-1}$

SO, $|\mathcal{E}|_{MAX} = N|B||A|\omega = 10^2 (10^{-1}\,T)(10^{-3}\,m^2)\left(10^3 \times \frac{\pi}{180}\,s^{-1}\right) = \frac{\pi}{18}\,V = \underline{0.2\,V}$

(d) THE SUGGESTED MODIFICATION LOOKS LIKE THIS (WHEN VIEWED FROM ABOVE):



IT DOESN'T MATTER HOW WE DRAG THE LOOP, THERE IS IDENTICAL FLUX $\oint \vec{B} \cdot d\vec{A} = |B||A|$ ($\cos\theta = \cos 0 = 1$), AND THE FLUX IS TIME-INDEPENDENT ($\mathcal{E} = -\frac{d\Phi_M}{dt} = 0$)

$\Rightarrow$ FARADAY'S LAW IMPLIES THAT ZERO EMF IS INDUCED IN THE LOOP.