A MOTOR CONTAINS A LOOP THAT IS TURNING THROUGH A UNIFORM MAGNETIC FIELD. A LOOP TURNING THROUGH A UNIFORM MAGNETIC FIELD IS ALSO HOW A GENERATOR WORKS, AS THE CHANGING FLUX THROUGH THE LOOP ELECTROMAGNETICALLY INDUCES A CURRENT IN THE LOOP. FOR A MOTOR, THE INDUCED EMF IS UNDESIRABLE AS IT FIGHTS THE EMF THAT IS DRIVING THE MOTOR. FOR THIS REASON IT IS CALLED A "BACK" EMF. THE PURPOSE OF THIS QUESTION IS TO MODEL SUCH A BACK EMF.

A COIL CONSISTING OF N LOOPS OF AREA A TURNS AT A RATE OF ω DEGREES PER SECOND THROUGH A UNIFORM MAGNETIC FIELD OF STRENGTH B. THE LOOP IS ORIENTED SO THAT AT $\theta = 0°$ THE NORMAL VECTOR TO THE LOOP POINTS IN THE SAME DIRECTION AS THE B-FIELD AND AT $\theta = 90°$ THE NORMAL VECTOR TO THE LOOP POINTS AT 90° TO THE B-FIELD. (a) FIND AN EXPRESSION FOR HOW THE AREA THROUGH WHICH THE B-FIELD PASSES CHANGES WITH TIME, GIVEN THAT THE B-FIELD PASSES THROUGH THE ENTIRE AREA AT $t = 0s$ ($\theta = 0°$) AND THROUGH NONE OF THE AREA AT $\theta = 90°$. YOUR EXPRESSION SHOULD BE IN TERMS OF A, ω AND t WHERE $\theta = \omega t$ (i.e. WHERE THE ANGLE $\theta = 0°$ AT $t = 0$ AND $\theta = 360°$ AT TIME $t = \frac{360°}{\omega}$) (b) HENCE DERIVE AN EXPRESSION FOR THE INDUCED EMF (THE BACK EMF IN A MOTOR) IN TERMS OF A, B, N, ω AND t (c) A MOTOR CONSISTS OF 100 LOOPS OF AREA $10\ cm^2$ TURNING THROUGH A 0.1 T B-FIELD AT A RATE OF 1000° PER SECOND. WHAT IS THE MAXIMUM BACK EMF INDUCED IN THE MOTOR? (d) A MODIFICATION TO THE MOTOR IS SUGGESTED WHERE THE COIL DOES NOT ROTATE, BUT IS INSTEAD HELD AT $\theta = 0°$ (SO THE B-FIELD PASSES THROUGH THE ENTIRE AREA) AND MOVED LEFT, RIGHT, UP OR DOWN AT A VELOCITY $\vec{v} = 1000\ m/s$. WHAT EMF WOULD BE INDUCED IN THE COIL FOR THIS SET-UP?

(a) (i) THE AREA THROUGH WHICH THE B-FIELD PASSES IS $|A|$ AT $\theta = 0°$ AND 0 AT $\theta = 90°$. SKETCH THIS SITUATION, TAKING CARE TO SKETCH THE DIRECTION OF $\vec{B}$ AND THE DIRECTION OF $\vec{A}$ (THE AREA NORMAL VECTOR) AS θ (THE ANGLE BETWEEN $\vec{A}$ AND $\vec{B}$) CHANGES. (ii) CAN YOU SEE THAT THE AREA WILL BE $-|A|$ AT $\theta = 180°$ AND 0 AT $\theta = 270°$? (iii) WHAT FUNCTION FOR A OBEYS $A(\theta = 0°) = |A|$, $A(\theta = 90°) = 0$, $A(\theta = 180°) = -|A|$, $A(\theta = 270°) = 0$? (iv) SUBSTITUTE IN FOR $\theta = \omega t$ IN YOUR EXPRESSION FOR $A(\theta)$.

(b) (i) TAKING YOUR EXPRESSION FOR $A(t)$ FROM PART (a) WRITE DOWN AN EXPRESSION FOR ϕ_M, THE MAGNETIC FLUX (i.e. THE AREA THROUGH WHICH $\vec{B}$ PASSES MULTIPLIED BY $|B|$). (ii) TO CHECK YOUR ANSWER WITH MORE FORMAL MATH YOU COULD USE THE DOT PRODUCT TO WRITE DOWN $\vec{B}, d\vec{A}$ IN TERMS OF θ (THE ANGLE BETWEEN $\vec{B}$ AND $d\vec{A}$) AND THEN INTEGRATE OVER $|dA|$ TO OBTAIN $\phi_M = \int \vec{B} \cdot d\vec{A}$ (iii) EITHER WAY, YOU HAVE AN EQUATION FOR $\phi(t)$... FARADAY'S LAW STATES $\mathcal{E} = -\frac{d\phi_M}{dt}$, SO WHAT IS THE VALUE FOR $|\mathcal{E}|$, THE MAGNITUDE OF THE BACK EMF? (iv) YOU MAY HAVE DERIVED AN EXPRESSION FOR THE FLUX THROUGH ONE LOOP... HOW WOULD THE TOTAL AREA, AND SO THE TOTAL FLUX, AND SO YOUR EXPRESSION, CHANGE FOR N LOOPS (i.e. FOR A COIL)

(c) (i) ALGEBRAICALLY, WHAT IS THE MAXIMUM POSSIBLE VALUE FOR $\mathcal{E}$, BASED ON THE EQUATION YOU DERIVED IN PART (b)? (ii) SUBSTITUTE IN THE NUMBERS, TAKING CARE TO REMEMBER THAT ω IS GIVEN IN DEGREES/SEC BUT THE SI UNIT IS RADIANS/SEC

(d) (i) SKETCH THE DESCRIBED SET-UP FROM ABOVE, DEPICTING THE DIRECTIONS OF $\vec{A}$, $\vec{B}$ AND $\vec{v}$ (ii) HOW DOES $\oint \vec{B} \cdot d\vec{A}$ CHANGE WITH TIME? (iii) SO, WHAT IS $\frac{d\phi_M}{dt}$? (iv) SO, WHAT IS $\mathcal{E} = -\frac{d\phi_M}{dt}$?