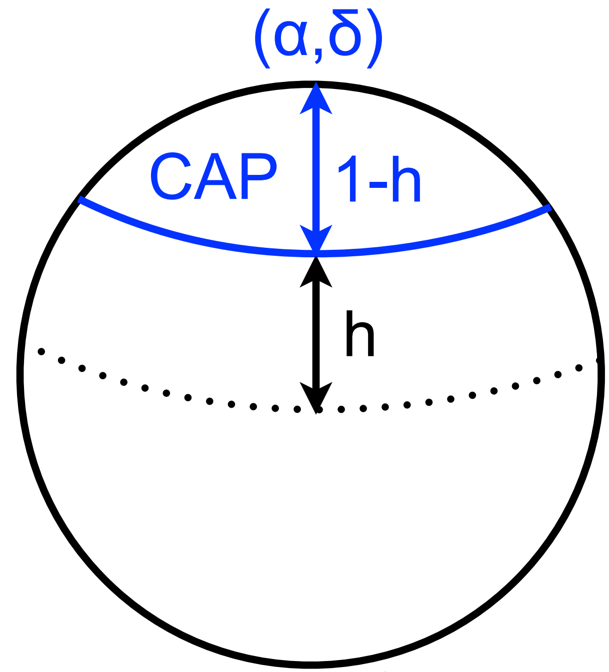
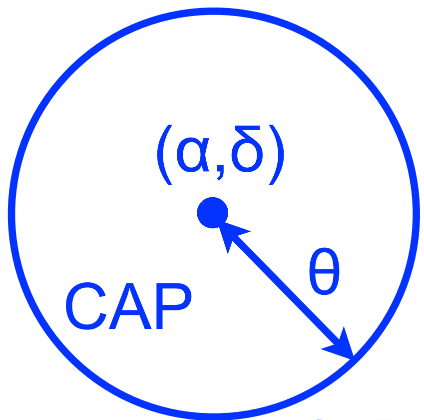
Spherical Caps

The Spherical Cap

- Spherical caps are useful models for representing arbitrary regions on the surface of the sphere
- A spherical cap is centered at a specific, Right Ascension and declination (α,δ)
- The height (1-h) of a spherical cap corresponds to some radius (θ) on the surface of the unit sphere
- By intersecting *multiple* spherical caps it is possible to construct general shapes on the surface of the sphere

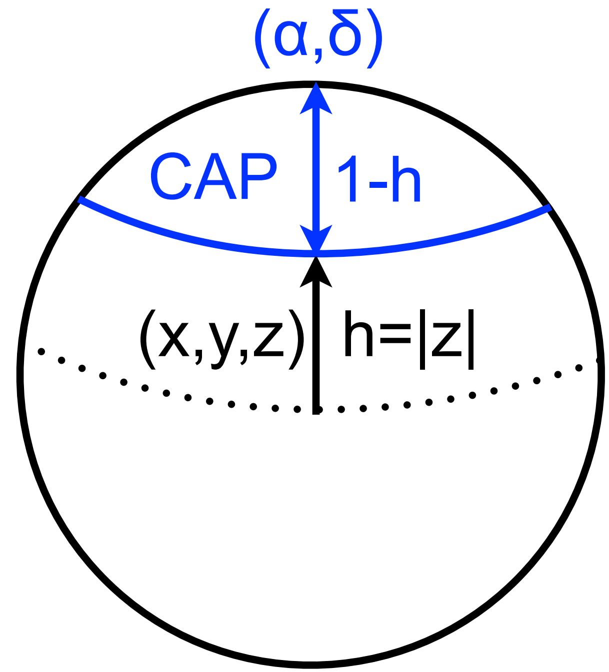


3-D view



2-D view

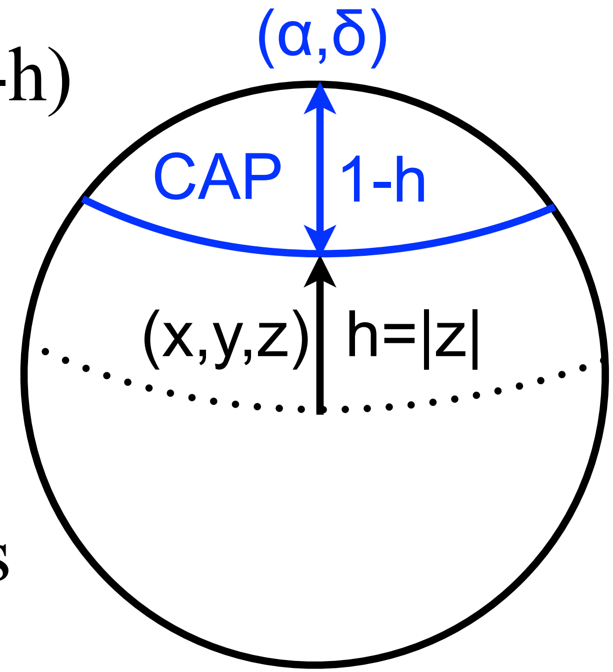
Vectorial representation of the spherical cap

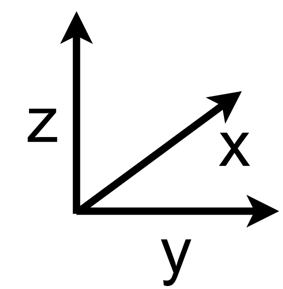


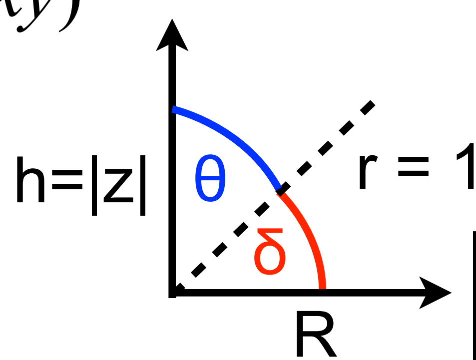
- As we know, spherical coordinates can be represented in Cartesian form (x,y,z) by the vector that points in the direction of (α,δ)
- Previously, for a *point on the surface of the sphere* we noted that $x^2 + y^2 + z^2 = 1$
- For a cap instead of a point, the *size* of the cap (which controls θ, the radius drawn on the surface of the sphere) can be controlled by $x^2 + y^2 + z^2 = h^2$
- Thus, one simple way to represent a cap is the 4-array given by x, y, z and h...we will choose the convention (x,y,z,1-h) as it is used by *mangle* (next week's focus)

Vectorial representation of the spherical cap

- Caps can be represented by (x,y,z,1-h)
- (x,y,z) is easy to determine, it's just the Cartesian conversion from (α,δ) (e.g., using *SkyCoord*)
- For astronomers, the natural way to think about the cap size is the radius drawn on the surface of the sphere (θ from the first slide, the radius we would put in *search_around_sky*)
- As shown in the diagrams to the right, 1-h = 1-cosθ
- So, (x,y,z,1-h) ≡ (x,y,z,1-cosθ)

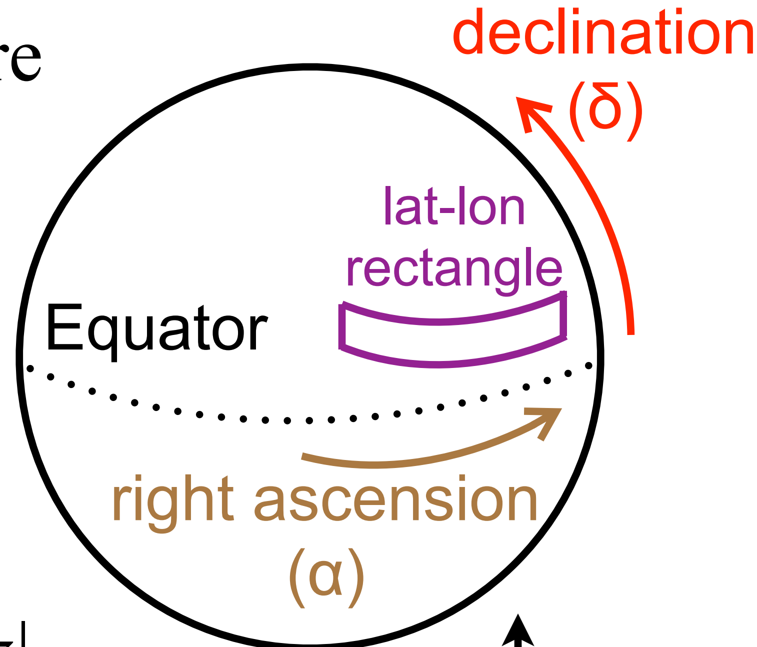


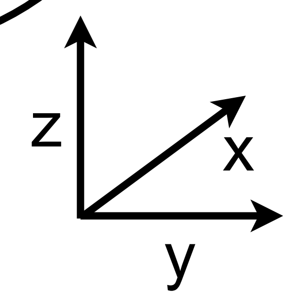


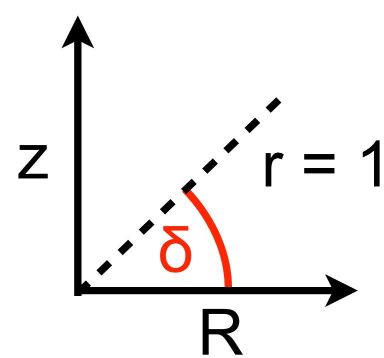


The area of a spherical cap

- As we discussed in the last lecture the area of a *lat-lon rectangle* is $2\pi(z_2 - z_1)$ where
  - $z_2 - z_1 = \sin\delta_2 - \sin\delta_1$
- For a spherical cap $\delta_2 = 90^\circ$ and the area is then $2\pi(1 - z_1)$
- But z_1 here is just what I called $|z|$ or h in previous slides
- So, the area of a spherical cap, $2\pi(1 - h)$ is easy to determine from the vector form for a cap
  - which is (x,y,z,1-h)



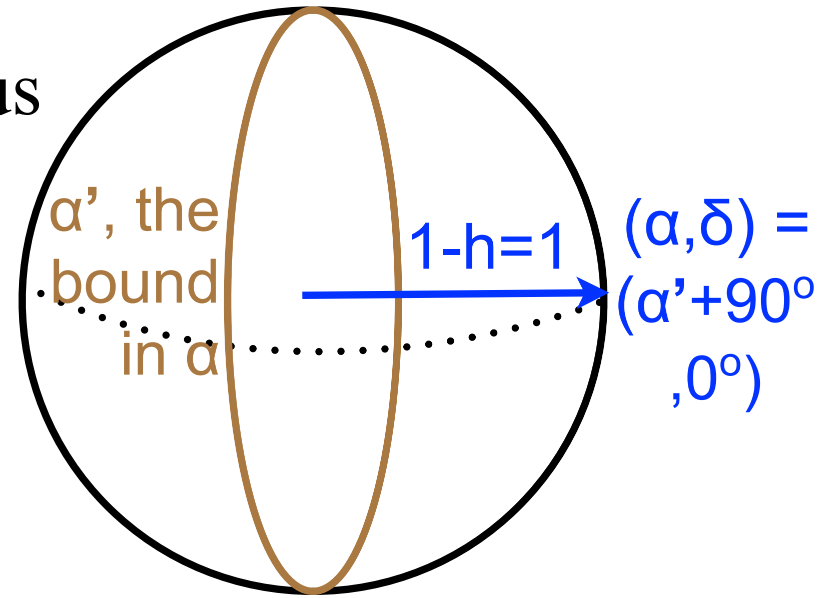




$z = r\sin\delta$
$= \sin\delta$

Caps bounded by Right Ascension

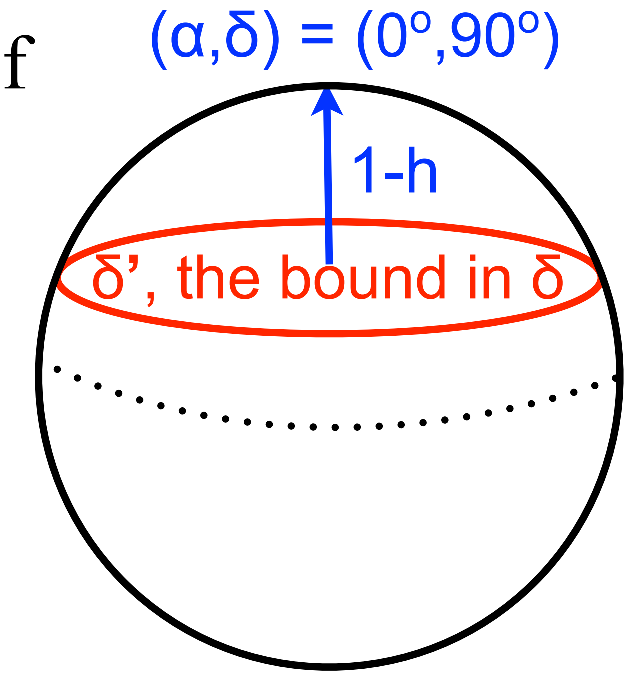
- So far we've discussed the general form, a "circle of radius θ on the surface of the sphere"
- Other main astronomy uses are fields bound by RA or dec
- Bounds in RA (α') map out a great circle on the sphere…a spherical cap that slices off exactly half of the sphere

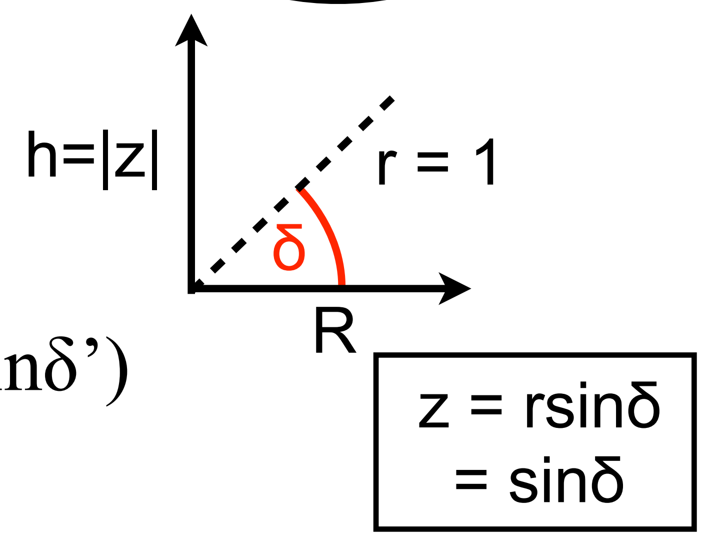


- So, the vector representation of a bound in RA is
  - $(x,y,z,1\text{-}h) = (xyz(\alpha'+90^o,0^o),\ 1)$
  - By *xyz()* I mean "conversion to Cartesian coordinates"

Caps bounded by Declination

- Bounds in dec (δ') map out lines of constant latitude on the sphere...a cap that slices off increasingly less of the sphere as δ' increases
- The (x,y,z) vector direction is always towards the north pole
- The size of the cap is given by $h = \sin\delta$
- So, the vector representation of a bound in declination is
  - $(x,y,z,1\text{-}h) = (xyz(0^o,90^o),\ 1\text{-}\sin\delta')$





Python tasks

1. Write a function to create the vector 4-array for the spherical cap bounded by 5^h in Right Ascension
   - the answer is [-0.96592582629, 0.25881904510, 0, 1]
2. Write a function to create the vector 4-array for the spherical cap bounded by 36^oN in declination
   - the answer is [0, 0, 1, 0.41221474770752686]
3. Write a function to create the vector 4-array for the spherical cap that represents a circular field drawn on the surface of the sphere at $(\alpha,\delta) = (5^h,36^o\text{N})$ with a radius of $\theta = 1^o$
   - the answer is [0.20938900596, 0.78145040877, 0.58778525229,0.00015230484]

Python tasks

4. Write a function that outputs your three spherical caps to a file in the following format:

```
1 polygon
polygon 1 ( 3 caps, 1 weight, 0 pixel, 0 str):
  -0.96592582629 0.25881904510 0 1
  0 0 1 0.4122147477075268 6
  0.209389006 0.781450409 0.587785253 0.00015230
```

But, make sure that the Cartesian vectors that define each cap are written out with **16** decimal places

The reason for this formatting should become clear next week